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EXPLORING THE FAMA-FRENCH FIVE FACTOR MODEL WITHIN A TIME-VARYING PARAMETERS FRAMEWORK

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Abstract. This paper evaluates whether time-variable parameters are present in the Five-Factor Model. The presence of time-dependent parameters may impact the asset pricing mechanism at the very core of the model. To investigate this assumption, we employed the global-local shrinkage priors in the Time-Varying Parameter models approach. We tested five key industry portfolios, covering a time window from July of 1963 to June 2022, obtaining 708 monthly observations. The results suggest that time dependency is present at the level of parameters. Furthermore, the pricing capability of the pricing factors is influenced by industry specifics while macroeconomic shocks impact various industries.

1. Introduction

The constant evolution and improvement of asset pricing models in the past decades represents an important milestone for practitioners and theoreticians alike. According to Kaya (2021), the cornerstone of such models appeared in the seminal work of Sharpe in the form of the so-called Capital Asset Pricing Model, which represents the basis on which various other asset pricing models are built. The main key takeaway of CAPM is its basis which is heavily influenced by the works of Markowitz concerning portfolio management and mean-variance optimization (Fisher and Statman, 1997). Thus, both upward and downward movements of the market and volatility can be viewed as a source of potential risk.

Hence, it can be argued that the model does not clearly distinguish between the concepts of risk and uncertainty. Consequently, the popularity and relevant application of factor-based models encouraged theoreticians to constantly improve existing models or build new ones that could better capture variations of excess return. Another relevant evolution concerning pricing models comes form of the so-called Arbitrage Pricing Theory which was developed by Ross (1976). The main advantages that APT has over the more traditional CAPM are fewer restrictions and model assumptions on the one hand, while on the other hand, it allows the addition of more risk factors. Thus, providing a relevant framework for the development of extended and more advanced factor-based asset pricing models.

Consequently, a similar approach was taken concerning the construction of both pricing models developed by Fama and French (1995; 2015). It can be generally argued that the market risk factor can be considered as a borrowed element from the traditional CAPM. According to Bhatnagar and Ramlogan (2012), relevant factors employed in the subsequent models, such as the value risk premium, the profitability and investment risk premium, or the size premium can provide the user with a better understanding of pricing mechanisms and risk exposures. Thus, the factor-based sort is the most important asset pricing model that ought to be further

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discussed in the paper. As a consequence, the remainder of the paper is dedicated to the evolution of this category of asset pricing models and the 5-Factor Model. It is also important to note that factor selection and testing occupy a large portion of the literature surrounding asset pricing models. According to Fama and French (2018), the popular testing methods for factor relevance may lead to a large level of "data dredging", and as a consequence, a large number of competing models would appear. This in turn may lead researchers to engage themselves in a "race" to develop the best model. Fama and French (2018) apply a set of tests to the CAPM, 3-Factor Model, 5-Factor Model, and an augmented variant incorporating a momentum factor. The results strongly favor the vanilla 5-Factor Model over all the aforementioned models, including the augmented variant. As a direct consequence of the tests and results provided, the authors suggest that a high degree of discipline is required in the field of asset pricing models. Considering that factors should have a strong theoretical background before their addition to the model.

This paper proposes a twofold contribution to the existing literature. Firstly, it tests the time-varying effects within the parameters and factors of the model. Secondly, it gauges to which extent such effects could impact different pricing and policy decisions and implications. Thus, we argue that such an approach regarding this particular model could provide the existing literature with a noteworthy contribution concerning the existing models. At the same time, it contributes to the existing methods and methodologies with a time-varying approach that could generate potential benefits and effects for different policy decisions in general and the asset pricing process, especially when considering the time-varying effects that impact the factor loading of pricing models.

Hence, such a methodology could provide a more accurate estimation of coefficients. At the same time, other close competitors such as the simple yet easy-to-implement rolling regressions approach fail to obtain unbiased and accurate coefficient estimates leading to suboptimal economic decisions and implications derived from them. Another aspect that favors the time-varying approach is that the rolling regression method is sensitive to the size of the observation space and generally tends to allow the flow of spurious non-linear patterns from the previous rolling window to the next. Hence, as a practical example, the rolling window approach may allow the flow of certain effects from one rolling window to the next, as argued in the work of Alptekin et al, 2019. Therefore, there is the risk that the time-varying effects of exogenous and endogenous shocks may flow from the crisis phase into the recovery phase. Consequently, providing biased and imprecise coefficients and results ultimately leading to poor policy or pricing decisions. The paper is divided into six sections. Section 2 is dedicated to the theoretical considerations regarding the 5-Factor Model. Section 3 discusses the time-variable effects of the 5-Factor Model while also showcasing the data employed and the normality tests conducted. Section 4 deals with the research methodology and data employed while Section 5 deals with the results and their interpretation. Section 6 is dedicated to a discussion concerning the implications for the selected industries, the general policy implications, and the model itself. The final section concludes the paper.

2. The 5-Factor Model

It can be considered that the roots of pricing models can be pinpointed to the seminal work of Markowitz, who introduced the concept of diversification while also demonstrating the impact of risk exposure on expected return. Furthermore, it should be noted that the seminal work of Markowitz adheres to the neoclassical finance theory. Such theoretical considerations regarding the neoclassical theory and the risk-neutral pricing concept can be generally considered as a foundation for the creation of different asset pricing models. Therefore, the aforementioned theoretical background and considerations can be traced to all the mainstream models starting with the CAPM and moving to finer refinements such as the 3 and the 5-Factor Models. Nevertheless, both models that were successfully developed by Fama and French (1992; 2015), consider that the market "beta" is a proper measure to evaluate volatility but may not be able to properly account for all the risk factors that may influence the price of an asset.

Therefore, as Hawaldar (2011) notes, the incomplete risk assessment provided by the market "beta" can be a starting point for future asset pricing models that aim to present a more rounded risk assessment tool that should account for all the different risk factors. As a consequence, we argue that Fama and French (1992; 2015), built both models with roots that originate within the CAPM. The latter is subordinated to the Modern Portfolio Theory therefore making no difference between the concept of risk and uncertainty. Furthermore, the works of Fama and French (1992; 2015) attempt to prove that the expected return is not dependent solely on price volatility. Therefore, while their models take into account the market risk factor of CAPM, the additional risk factors employed show that other elements of potential risk can exert a degree of influence on the return. Given the performance of the 3-Factor Model, a further and natural evolution of this model is provided in the form of the 5-Factor Model. This improved model attempts to build upon the potential provided within the 3-Factor model by incorporating 2 additional risk factors or variables. The newly added factors that are considered to be potential sources of risk are the Profitability and Investment risk factors. Thus, the augmentation of the 3-Factor Model with the new factors as mentioned earlier results in the 5-Factor Model:

$$R_{it} - R_{ft} = \alpha_{it} + \beta_1(R_{Mt} - R_{ft}) + \beta_2 SMB_t + \beta_3 HML_t + \beta_4 RMW_t + \beta_5 CMA_t + \epsilon_{it} \quad (1)$$

Here, Rit represents the return of a portfolio i at time t, and Rft represents the return obtained by holding a safe asset such as a government bond or treasury. The difference between the two variables represents the excess return obtainable by holding a portfolio of stocks. RMt represents the total return of the market while RMt-Rft represents the excess return of the market against the safe asset. The remaining factors are the SMB factor, which represents the size risk exposure and is computed as Small caps minus Large caps (Small minus Big), the HML factor (High minus Low), or the value factor, which represents the risk exposure arising from holding a high book-to-market portfolio. While RMW (Robust minus Weak) represents the profitability risk factor, and CMA (Conservative minus Aggressive) or the investment risk factor is added. Following a similar empirical approach, the factors are computed as the difference between portfolios composed of Robust earnings companies minus Weak earnings companies in the case of the RMW, or the profitability factor. While CMA or the investment factor aims to present the difference in return between companies with a conservative or aggressive investment strategy.

Given the aforementioned criticism regarding the 3-Factor Model, coupled with the fact that additional risk factors could improve the pricing ability of the model, Fama and French (2015) decided to augment the 3-Factor Model with the 5-Factor Model. According to Douagi et al (2021) and Liammukda (2020), the performance of the 5-Factor Model against the 3-Factor Model can be noted when, for instance, the models are implemented for the equity markets of both Australia and Japan. The main findings regard the 5-Factor Model as a better predictor concerning asset pricing when directly compared to the simpler 3-Factor Model. It can be argued that this result is due to the additional Investment and Profitability risk factors. An interesting finding has nevertheless been reported by Douagi et al (2021) when investigating the pricing capability of the 5-Factor Model on the Australian equity market. Namely, the general liquidity or illiquidity of stocks can be successfully used and incorporated as a risk factor. This new Liquidity risk factor can provide two relevant insights: on the one hand, the pricing power of the 5-Factor Model can be augmented by adding different custom-tailored risk factors that can prove to be more relevant for the market in question.

The 5-Factor Model in its vanilla form may yield unsatisfactory results when applied to a less developed and less efficient capital market. Thus, the 5-Factor Model is indeed an upgrade concerning the pricing power and relevance of the 3-Factor Model. The model can provide practitioners and theoreticians with a powerful tool that can be used in both developed and emerging markets. Nevertheless, the best and most relevant results are obtained when the model is employed in developed markets while its implementation in frontier and emerging markets exposes several shortcomings.

The 5-Factor Model of Fama and French (2015), can be viewed as an upgrade or extension of their previously working model. According to Kaya (2021) and Foye and Valentinčič (2020), this new model is extended by adding 2 new risk factors, CMA, which represents the investment risk premium, and RMW which attempts to represent the risk premium regarding the profitability of the company. The addition of the new factors significantly improves the pricing capability of the model, especially when compared to the pricing power of the 3-Factor Model. According to Liammukda et al (2020), the results obtained after applying the 5-Factor Model in the case of Asian stocks have outperformed the more traditional asset pricing models.

Nevertheless, as the authors argue, it is important to highlight certain differences that have been identified with the occasion of this study. On the one hand, in the case of Chinese stocks the profitability risk premium, RMW, proved to be the most effective factor in explaining average returns. While on the other hand, the results obtained after applying the 5-Factor Model on the Jakarta Stock Exchange provide a different explanation. Namely, the CMA or investment risk premium proved to be the most relevant factor in explaining average returns. While RMW or the profitability risk premium, proved to be the most effective factor in the case of Chinese stocks, it became insignificant in the Indonesian scenario.

Another relevant aspect obtained in this analysis regards the size risk premium SMB, which in turn exerted a negative and significant size effect on return. Given this, one may note that the 5-Factor Model has several limitations, especially when the original or vanilla model with the basic factor selection is applied either to different geographical locations or more importantly, to different markets. The latter element is the most relevant, as emerging markets generally suffer from one form or another of informational inefficiency or, in some cases institutional inefficiency (Foye and Valentinčič, 2020). Another dilemma arises from the fact that the addition of the RMW and CMA risk factors improves the overall predictive power of the model.

As a consequence, the inclusion of these factors results in the value factor HML, becoming dispensable (Fama and French, 2015, Hou et al, 2019). Another favorite criticism of the model arises from the fact that similar to the traditional asset pricing models, the 5-Factor Model is based on the assumption of market efficiency and on the fact that investors are rationally managing their portfolios. While in the same time, the linear risk and return relationship is considered to provide a normal distribution of asset returns (Jan and Ayub, 2019). Another limitation of the model is that it ignores price and earnings momentum. This implies that the model violates the time-varying expected returns and tends to oversimplify the general economic environment by applying a linear regression model with a constant intercept and constant slopes (Hou et al, 2019).

This limitation has the potential to underrepresent the variations and effects that different risk factors can exert over time. In consequence, as Liammukda et al (2020) note, the limitations of both models in their vanilla form tend to pose significant hurdles in pricing capability. This limitation can be observed in the case of emerging markets where a volatile and unpredictable economic environment has the potential to limit the efficiency of the chosen variables within the model. Another critical aspect regarding the violation of the time-varying notion is that during the entire holding period, the expected return of a portfolio remains constant (Hou et al, 2019). Thus, one can argue that the autoregressive construction of most pricing models employs fixed values for each parameter and usually requires the data employed to be stationary (Hongsakulvasu and Liammukda, 2020).

Given this, it is important to argue that in the case of the 5-Factor Model, such limitations may hamper its predictive power and that this issue may have a larger impact on its application in emerging or frontier capital markets. The reason for this, as proved by Liammukda et al. (2021) and Hou et al. (2019), is that volatility in general and risk transmission in particular, are more distinct in such markets while the effect of the variables of the 5-Factor Model can have wide fluctuations over time. Therefore, the effects of the variables employed may affect the overall pricing capability of the model.

3. Time Variability of the 5-Factor Model

This section of the paper is entirely dedicated to the testing of the factors found within the 5-Factor Model in a time-varying parameters (TVP henceforth) framework. Given the construction of the factors, the objective of this section revolves around testing the time variability hypotheses of the factors employed by the 5-Factor Model. Testing the factors in such a manner may provide valuable insight concerning the impact of the factors given different endogenous and exogenous shocks. To this end, and to better present such evolutions, the observation period covers a large variety of events and shocks, both exogenous and endogenous.

Thus, the choice of this econometric tool was motivated by the fact that this model can display not only the effects of the explanatory variables, namely the factors on the outcome variable but can also account for their evolution over time (Cadonna et al, 2020). Nevertheless, the main issue of any TVP model is represented by the fact that employing a data set that contains a large number of variables could result in the overfitting effect. Hence, directly and negatively impacting the predictive power of the model and providing mixed results (Knaus et al, 2021; Cadonna et al, 2020; Bitto and Schnatter, 2019). Providing an insight into the overfitting issue Cadonna et al (2020) and Bitto and Schnatter (2019), argue that a larger number of priors such as the Triple Gamma prior, the Double Gamma prior, or the Bayesian Lasso prior have a positive influence in eliminating the overfitting effect.

Another important effect is that a model that uses different priors is more accurate in capturing the time variability of a large number of factors. Given the size of our observation period, 708 monthly observations, such a model could prove extremely efficient. To this end, we employ the "shrinkTVP" R package developed by Knaus et al. (2021) and further improved and explored by Knaus et al. (2022).

This package provides the tools that are represented by a working model that uses shrinkage priors to reduce noise and most crucially, to remove the risk of overfitting altogether. The resulting models are the simple Bayesian Lasso, the Bayesian Lasso with ridge prior followed by the Double and Triple Gamma priors. Another important addition comes in the form of the Through Stochastic Volatility model, which excels at capturing time variability, especially in the case of asset prices.

Nevertheless, it can be argued that the TVPM-S methodology also presents forecasting potential. To this end, we argue that the ability of the priors to differentiate between time-varying and static coefficients is a significant advantage. As Bashir and Usman (2020), employing the TVPM-S methodology to forecast inflation provides reliable coefficients and forecasting results while at the same time providing relevant economic insights that may aid policymakers. Especially given the existence of the stochastic prior that excels at forecasting both volatility and inflation.

Another advantage is presented in the work of Gächter et al (2023) who observed GDP growth over a large time horizon within the growth-at-risk (GaR) literature. The main concept of GaR revolves around the investigation of potential downside risks and deteriorating conditions that may hamper economic growth, in a similar fashion to the value-at-risk concept operated in finance. It is important to note that this particular area of research deals with very large time horizons. Thus, as a consequence, the aforementioned author decided to implement a TVPM-S methodology which presents a crucial advantage for the task it was set upon. Namely, it can accurately identify time-variability amongst a large plethora of factors and can also be employed to identify potential structural breakpoints that generally plague data sets with large time horizons.

Thus, the main advantage of this methodology lies in the implementation of a large number of priors, especially the stochastic prior, which, as previously noted, can forecast inflation and also volatility in a satisfactory manner. Moreover, as the aforementioned authors note, the time-varying methodology also allows for clear identification of structural breakpoints which are generally present within data sets with a large number of observations and large time horizons.

Given such a large number of empirical tests and methodological advantages and the fact that the variables we aim to test are significant in number and cover a large time horizon, the choice of this model was favored. Hence, we will further proceed by presenting the data and methodology employed followed by the section containing the results obtained after applying various specifications of the model.

3.1. **Data.** Given that we aim to test whether the parameters of the 5-Factor Model vary over time and to which degree, while employing the TVPM-S methodology, the choice of data is understandable. With this aim, we have used the main factors of the model which are computed similarly to the work of Fama and French (2015).

Hence, the factors employed in the model are: High minus Low (HML), or the value factor, Small minus Big (SMB), or the size factor, Robust minus Weak (RMW), or the profitability factor, Conservative minus Aggressive (CMA), or the investment factor. Lastly, a reminder of the fact that both models are linked to the classical CAPM, the market risk factor namely, the excess return generated by the market. These factors have been constructed under a 2x3 sort according to the framework of Fama and French (2015).

In this type of sort, the stocks are firstly divided by size with their B/M ratios. Thus, Fama and French (2015) divide the stocks into 6 subcategories namely: Small Value, Small Neutral, and Small Growth for the Small size category of stocks. The Big category is constructed as follows: Big Value, Big Neutral, and Big Growth. A similar approach is taken during the next 2 sorts, based on Size and Operating Profitability (RMW) and, lastly by Size and Investment (CMA). After the sorts on Size and B/M, Size and Profitability, and Size and Investment have been computed, the Size (SMB) factor is the average of the 3 sorts mentioned above. The mathematical descriptions of the process are the following:

$$\begin{aligned}\mathrm{SMB}_{B/M} =& \frac{\text{Small Value} + \text{Small Neutral} + \text{Small Growth}}{3} \\ &- \frac{\text{Big Value} + \text{Big Neutral} + \text{Big Growth}}{3}\end{aligned} \tag{2}$$

$$\begin{aligned}\mathrm{SMB}_{OP} =& \frac{\text{Small Robust} + \text{Small Neutral} + \text{Small Weak}}{3} \\ &- \frac{\text{Big Robust} + \text{Big Neutral} + \text{Big Weak}}{3}\end{aligned} \tag{3}$$

$$\begin{aligned}\mathrm{SMB}_{INV} =& \frac{\text{Small Conservative} + \text{Small Neutral} + \text{Small Aggressive}}{3} \\ &- \frac{\text{Big Conservative} + \text{Big Neutral} + \text{Big Aggressive}}{3}\end{aligned} \tag{4}$$

$$\mathrm{SMB} = \frac{\mathrm{SMB}_{B/M} + \mathrm{SMB}_{OP} + \mathrm{SMB}_{INV}}{3} \tag{5}$$

Following this initial sort, the stocks are further screened and divided by the following factors: HML, RMW, and CMA. The sorting of these remaining factors is done similarly. For the Value factor (HML), the factor is computed as the average return of the Value portfolios minus the average return of the Growth portfolios. For the Profitability factor (RMW), the value of the factor is the average between the return of the Robust earning portfolios minus the Weak earnings portfolios. Lastly, in the case of the investment factor (CMA), the value of the factor results from the average return of the portfolios containing stocks with a Conservative approach minus the average return of the portfolios containing stocks with an Aggressive investment approach. The mathematical expressions are as follows:

$$\text{HML} = \frac{\text{Small Value} + \text{Big Value}}{2} - \frac{\text{Small Growth} + \text{Big Growth}}{2} \tag{6}$$

$$\text{RMW} = \frac{\text{Small Robust} + \text{Big Robust}}{2} - \frac{\text{Small Weak} + \text{Big Weak}}{2} \tag{7}$$

$$\text{CMA} = \frac{\text{Small Conservative} + \text{Big Conservative}}{2} - \frac{\text{Small Aggressive} + \text{Big Aggressive}}{2} \tag{8}$$

Given this, the last factor namely the excess return of the market, follows a classical framework, the monthly return on stocks is deducted from the return on the safe asset. In our case, the safe asset is represented by the return of the U.S. 1 Month Treasury Bill. With this in mind, the right-hand side of the 5-Factor Model is accounted for, but to test the effects and the time-varying of the factors the left part of the equation is required. To complete the equation, the excess return of a select 5 industries has been chosen. The excess return has been computed classically, namely, the monthly return of each Industry, minus the return provided by our safe asset, the 1 Month Treasury Bill. To test the variable effects of the factors, we employ the initial key industries provided by Fama and French (2015). This approach regarding a reduced but relevant dataset is also followed in the existing literature, such as the case of Yan and Bao (2020), who test the effects exclusively within the "Manufacturing" and "Health" Industries. We argue that this initial selection may provide a perfect dataset for the TVPM-S methodology while at the same time, providing sufficient and relevant economic insight regarding the impact of the factors and the existing time-variable effects. Given the vanilla setup of the model, and the decision to test the 5-Factor Model in its most basic form, employing this initial selection of industries is more appropriate.

To this end, the industries selected are the "Consumer" industry (CNS), the "Manufacturing" industry (MAN), the "Technology" industry (TECH), the "Health" industry (HLT), and, lastly the "Other" industries (OTH). All the data employed is collected from the database provided by French (2023), on a monthly frequency. The observation window used begins in July 1963 and ends in June 2022. This results in a number of 708 monthly observations. The usage of monthly observations was chosen due to the application of the model in the case of short- or medium-term portfolio management. Another relevant aspect worth mentioning regards the types of companies or sectors of activity that are incorporated in the chosen industries. To this end, we will proceed to clarify and present the most relevant sectors by making use of the Standard Industrial Classification (SIC) codes.

Starting with the "Consumer" Industry, which is represented by companies that operate in the Agricultural, livestock, fishing, and trapping sectors with SIC codes from 100-999. Followed by companies that operate within the Wholesale Trade with both durable and non-durable goods, having the SIC codes from 5000-5199. Other relevant sectors are Food and Kindred Products, SIC code 2000-2099; Tobacco products, SIC code 2100-2199 and lastly, Apparel and Finished products, SIC code 2300-2399.

The next industry of interest is represented by the "Manufacturing" Industry. The key sectors within this industry are Industrial and Communication Machinery and Electronic and Electrical Equipment SIC codes 3580-3621 and 3623-3629. Followed by Transportation Equipment, SIC code 3700-3799.

A relevant sector in today's climate, the Energy sector has also been included. In this sector, we find Electric, Gas, and Sanitary Services, SIC code 4900-4999, and Petroleum Refining, SIC code 2900-2999.

Regarding the "Technology" Industry, the most relevant sectors are Computer Programming and Data Processing, SIC code 7370-7372; R&D labs SIC code 7391-7391; Computer Processing, Data Preparation and Processing, SIC code 7374-7374 and lastly, Research, Development, Testing Labs, SIC code 8730-8734.

The next industry of interest is the "Health" Industry, where the most relevant sectors are: Health and Associated Services, SIC code 8000-8099; Medical Equipment, SIC code 3841-3845 and lastly, Medicamentation and Similar Products, SIC code 2833-2834.

Lastly, the so-called "Other" Industry comprises sectors such as Mining, Construction, Building Materials, Entertainment and Finance. Therefore, the resulting key industries represent the entirety of stocks from the NYSE, AMEX, and NASDAQ, pertaining to the above sectors of activity. Which are further allocated to one of the five key industries employed. Therefore, a direct consequence of this varied selection is that it provides a proper testing environment for the TVPM-S methodology as the data employed is heterogeneous and allows for the testing of time-variable effects. Furthermore, an additional challenge to the selected methodology is the wide horizon of the data frame employed. This particular challenge, so to speak, stems from the economic developments that have occurred within the data frame. Especially when considering the various exogenous and endogenous shocks that have occurred and also the presence of potential structural breakpoints.

Before discussing the methodology employed, several normality tests have been conducted to better select an appropriate model. Several normality tests such as the Shapiro-Wilk (1965), Andersen-Darling (1954), Kormogolov-Smirnov (1951), and Jarque-Berra (1987) tests, have been used. The results obtained for the normality tests are organized in Table 1.

Results for the normality tests for each selected industry

Test	**Consumer**	**Manufacturing**	**Technology**	**Health**	**Other**
Shapiro	6.87×10^{-9}	8.659×10^{-11}	3.02×10^{-8}	9.52×10^{-8}	3.58×10^{-9}
Andersen-Darling	2.80×10^{-6}	3.253×10^{-10}	7.68×10^{-8}	3.402×10^{-4}	5.73×10^{-8}
Kolmogorov-Smirnov	0.1141	0.01389	0.0192	0.2062	0.05929
Jarque-Berra	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16	<2.2e-16
Anscombe	1.05×10^{-11}	4.27×10^{-11}	8.36×10^{-8}	8.29×10^{-11}	7.20×10^{-9}
Geary	0.7462	0.7403	0.7501	0.7573	0.7557
Bonnet-Seiter	2.04×10^{-11}	6.36×10^{-14}	6.45×10^{-10}	1.67×10^{-7}	5.25×10^{-8}
Kurtosis	5.5149	5.3696	4.6331	5.3022	4.8639
Skewness	-0.3236	-0.5288	-0.3685	-0.0074	-0.4937

Standard errors are reported in parentheses. *, **, *** indicates significance at the 90%, 95%, and 99% level, respectively.

After conducting several normality tests on the following industries: "Consumer", "Manufacturing", "Technology", "Health", and "Other" industries, we argue that the data employed displays a non-normal distribution. Nevertheless, the results provided by the Kolmogorov-Smirnov data distribution test which was pioneered and implemented by Massey (1951), suggest a normal distribution of the employed data. Hence, given the strong results obtained in the majority of tests employed, we decided to rule out the results of the Kolmogorov-Smirnov test. Another element that favors such an approach comes in the form of density plots and Quantile-Quantile plots.

Given the results obtained from both the formal normality tests and the graphical approach, we argue that the data employed follows a non-normal distribution. The main models employed are the generalized linear model (GLM), which uses fixed parameters, and the Time-Varying Parameter Model with Shrinkage (TVPM-S), which uses priors and employs dynamic parameters.

4. Methodology

This section of the paper covers the methodology employed to test the time-varying parameters of the 5-Factor Model factors. Thus, in the following lines, we will highlight the general specification employed in the model. The model is implemented within the "shrinkTVP" package in the R programming language (Knaus et al. 2021; 2022). The Time-Varying Parameter (TVP) specification can have the following general form:

$$\begin{aligned} y_t &= x_t\beta_t + \varepsilon_t \quad \varepsilon_t \sim N(0, \sigma_t^2), \\ \beta_t &= \beta_{t-1} + w_t \quad w_t \sim N_d(0, Q), \end{aligned} \tag{9}$$

Where, y_t is the univariate response while x_t represents the dimensional row vector which contains the regressors at time t, with x_{t1} being the intercept. Assuming that $Q = \mathrm{Diag}(\theta_1......\theta_d)$ represents a diagonal matrix considering state innovations as independent. While the initial values follow a normal distribution process with a starting mean $\beta = (\beta_1......\beta_n)$. Equation 9 can be used in a non-centered parametrization manner as follows:

$$y_t = x_t\beta + x_t\mathrm{Diag}(\sqrt{\theta_1}...\sqrt{\theta_d})\beta_t^2 + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_t^2),$$

$$\tilde{\beta}_t = \tilde{\beta}_{t-1} + \tilde{u}_t, \quad \tilde{u}_t \sim N_d(0, I_d), \tag{10}$$

with $\beta_0^2 \sim N_d(0, I_d)$, where I_d represents the d-dimensional matrix and $\tilde{u}_t$ accounts for the non-centered parametrization. The "shrinkTVP" package is also able to model the observation homoscedastically and heteroscedastically using stochastic volatility (SV) prior specification. In this particular case, the log-volatility $h_t = \log \sigma_t^2$, seen in the formula:

$$h_t|h_{t-1}, \mu, \phi, \sigma_\eta^2 \sim N(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2), \tag{11}$$

while the initial state of $h_0 \sim N(\mu + \phi(h_{t-1} - \mu), \sigma_\eta^2)$. The benefit of the stochastic volatility model applied to the error term has the advantage of avoiding the detection of mock variations that may appear in the model coefficients by capturing a certain degree of variability within the error term. Further details regarding the statistical modeling of the "shrinkTVP" package are presented in Knaus et al (2021). It is worth noting that the package runs the specifications using different priors to better capture the time-varying nature of the independent variables employed.

The priors are Hierarchical Bayesian Lasso, Hierarchical Bayesian Lasso with ridge prior, Hierarchical Bayesian Lasso with the Double Gamma prior, and the Hierarchical Bayesian Lasso with the Triple Gamma prior. Lastly, the package also generates a Through Stochastic Volatility model. Nevertheless, to this end, we consider that such an approach may provide valuable insight when applied to the factors of the model.

Given this, we argue that the Bayesian estimation approach utilized within the Time-Varying Parameter Models with Shrinkage (TVPM-S) methodology brings a couple of advantages. One such advantage is the number of priors employed which proves efficient in capturing the time-variable effects of the model. Another advantage of the priors is that they prevent the model from overfitting, especially considering the large number of variables employed. Lastly, we consider that this methodology suits our needs well, especially when attempting to test the time-variable effects using a dynamic parameters approach. Given this, the decision to test the time-variable effects of the 5-Factor Model in a static parameters model was taken. To this end, we have employed a simple GLM model within the R programming language.

The package employed is "biglm" (Lumley, 2022), which allows the usage of different families and links. An issue encountered in this particular model's case lies within the non-normal data distribution. To accommodate this development and to maintain a working model the GLM model created uses a quasi-distribution. The main advantages of the GLM model, lie in its ease of application, interpretation, and in its ability to minimize the overfitting effect. The general specification used for the aforementioned industries namely: "Consumer" (CNS), "Manufacturing" (MAN), "Technology" (TECH), "Health" (HLT), and lastly, the "Other" industries (OTH). The general specification employed within R is the following:

$$Er_t = \alpha + \beta_1 \text{Mkt.RF}_t + \beta_2 \text{SMB}_t + \beta_3 \text{HML}_t + \beta_4 \text{RMW}_t + \beta_5 \text{CMA}_t + \epsilon_t \tag{12}$$

Where:

- Er_t represents the monthly excess return of the given industry at time t,
- Mkt.RF_t represents the excess return provided by the entire market,
- SMB_t variable depicts the size factor,
- HML_t variable depicts the so-called "value" factor,
- RMW_t variable depicts the profitability factor,
- CMA_t variable depicts the investment factor,
- ϵ_t is the estimation error.

Another relevant aspect worth mentioning is that whenever a factor presents time variability, it is relevant in the asset pricing formation that the 5-Factor Model attempts to capture. In the case of no time variability effects, it can be argued that the factors fail to influence the asset price formation process. The results obtained will be presented in the following section.

5. Results

This section of the paper deals with the results obtained after applying the methodology presented in the previous section. For all scopes and purposes, the "shrinkTVP" package run in the R language yielded a number of 5 distinct models with different priors. The graphical representations of the results obtained can be viewed in the Appendices section. The main reason behind this approach lies in the fact that employing a large number of variables with a large observation window while using several priors implemented under different models should provide better and more reliable results. Thus, allowing us to observe the time-varying effect of the dependent variables.

With this in mind, the tables of coefficients and the graphical representations obtained from each model have been included to better present the time variability of the factors over the observation period. Another aspect worth mentioning is the fact that before applying the TVPM-S model, a simple Generalized Linear Model was employed using the R package "biglm" (Lumley, 2022). The choice of such a model was made to exhibit the effects of the model on the selected industries while employing both static and dynamic parameters. Another relevant aspect is that the main results are presented in a condensed format and structured in such a manner as to allow a better understanding of what caused and influenced such results.

Hence, the industries are discussed at large due to the nature of the results obtained and the exogenous and endogenous variables at play.

5.1. Results for the "Consumer" Industry.

The first industry analyzed is the "Consumer" Industry which yielded interesting results after applying all the aforementioned models.

As can be observed in Table 1, the results obtained after applying the GLM model, showcase the most relevant and statistically significant factors in the case of the "Consumer" Industry. It can be noted that, the Mkt.RF, SMB, and RMW factors are the most relevant while the HML factor loses relevance. We argue that the results obtained within the GLM framework are in line with the theoretical considerations involved and the results provided can serve as a comparison basis with the results provided by the shrinkTVP methodology.

To this end, we argue that, in the case of the "Consumer" Industry, the most relevant factors that can be identified in both the shrinkTVP and the GLM frameworks are the Mkt.RF and RMW factors. With this in mind, the first model in question is the Hierarchical Bayesian Lasso, the results of which can be found in Table 2. As can be observed in Table 2, the results indicate that the parameters in question are time-varying, the largest fluctuations over time can be observed in parameter Mkt.RF, with a beta of 1.07. This parameter is followed by the RMW parameter or factor, with a beta of 0.413. The remaining parameters, namely SMB,

Table 1. Results for the Generalized Linear Model for the "Consumer" Industry (CNS)

Variable	**Coefficient (Standard Error)**
Intercept	-0.112 (0.064)
Mkt.RF	0.981*** (0.016)
SMB	0.122*** (0.022)
HML	-0.036 (0.029)
RMW	0.443*** (0.030)
CMA	0.221*** (0.045)

Notes: Standard errors are reported in parentheses. *, **, *** indicates significance at the 90%, 95%, and 99% level, respectively. The results have been obtained by employing the `biglm` package in R. The package creates a generalized linear model that uses only p^2 memory for p variables. For further information regarding the model and its general implementation, see Lumley, Thomas (2020). *biglm: Bounded Memory Linear and Generalized Linear Models.* R package version 0.9-2.1., `https://CRAN.R-project.org/package=biglm`.

HML, CMA, and the Intercept do present a certain degree of time variation, albeit not as strong as the Mkt.RF and RMW parameters. This can be confirmed by the values of the posterior density.

The graphical representations of these results can be seen in Figure 1 and in Figure 2, where, it can be observed that all the parameters analyzed are time-varying. It is interesting to note the fact that the excess return of the market factor, Mkt.RF presents 2 major troughs, namely in the 1980s and the 2008-2010 period. It can be argued that during these major events, namely the stagflation which occurred in the 1980s and, the Global Financial Crisis (GFC henceforth) of 2008-2010, investors sought to move away from stocks. This behavior is typical during such situations when safe-haven assets such as government bonds take a leading role over stocks, which may be perceived as being riskier.

The peak in the case of the Mkt.RF factor can be traced to the period of the economic boom in the 1990s. Nevertheless, it should be noted that another decline can be traced back to the early 2000s when the then Dotcom Bubble collapsed. On the other hand, in the case of the Profitability factor, RMW, the time variable effects seem to maintain a uniform and constant evolution over time. Notable exceptions can be observed in the 1980s period and, a more generalized decline is shown to have occurred after the 2010s. This can be attributed to the negative economic outlook that succeeded the GFC. The next model applied to the "Consumer" Industry is represented by the Hierarchical Bayesian Lasso with Ridge prior. The results obtained from applying this model can be seen in Table 3.

Similar to the Hierarchical Bayesian Lasso, the parameter that has the highest time variation is yet again the Mkt.RF parameter, with a beta of 1.001, followed by HML with a beta of 0.061, and RMW with a beta of 0.41. Those results suggest that yet again the excess return of the market has the highest variability between the given set of parameters. This should not come as a surprise, given the fact that the market return and the return provided by the safe asset, namely the 1 Month T-Bill, have a dynamic evolution. While in the case of the latter, the

TABLE 2. Results of the Hierarchical Bayesian Lasso for the "Consumer" Industry

Parameter	Mean	SD	Median	HPD 2.5%	HPD 97.5%	ESS
β_2 Intercept	-0.082	0.117	-0.082	-0.335	0.150	1246
β_2 Mkt.RF	1.070	0.094	1.069	0.888	1.261	149
β_2 SMB	0.058	0.106	0.061	-0.168	0.249	194
β_2 HML	0.016	0.182	0.015	-0.347	0.378	277
β_2 RMW	0.413	0.132	0.414	0.111	0.659	323
β_2 CMA	0.011	0.167	0.015	-0.333	0.324	331
$\|\theta\|$ Intercept	0.011	0.010	0.008	0	0.032	830
$\|\theta\|$ Mkt.RF	0.013	0.005	0.013	0.004	0.024	349
$\|\theta\|$ SMB	0.015	0.008	0.015	0.002	0.030	378
$\|\theta\|$ HML	0.053	0.014	0.053	0.028	0.082	240
$\|\theta\|$ RMW	0.012	0.009	0.010	0	0.028	287
$\|\theta\|$ CMA	0.031	0.016	0.029	0.003	0.064	225
τ^2 Intercept	0.255	0.695	0.092	0	0.919	1206
τ^2 Mkt.RF	0.616	0.710	0.414	0.076	1.750	1157
τ^2 SMB	0.240	0.574	0.086	0	0.928	1222
τ^2 HML	0.273	0.701	0.099	0	1.014	1276
τ^2 RMW	0.374	0.703	0.197	0.001	1.236	1324
τ^2 CMA	0.272	0.799	0.098	0	0.982	1874
ξ^2 Intercept	0.002	0.004	0.001	0	0.009	1816
ξ^2 Mkt.RF	0.002	0.004	0.001	0	0.008	2020
ξ^2 SMB	0.002	0.005	0.001	0	0.009	2207
ξ^2 HML	0.004	0.005	0.002	0	0.012	1134
ξ^2 RMW	0.002	0.004	0.001	0	0.009	1728
ξ^2 CMA	0.003	0.004	0.002	0	0.010	1710
κ^2 B	963.545	698.950	3790.720	64.816	2352.265	979
λ^2 B	11.159	10.101	8.330	0.161	30.165	819
σ^2	1.840	0.117	1.836	1.618	2.069	998
C_0	1.924	0.705	1.840	0.657	3.256	5000

Notes: The parameters employed in the resulting models are as follows: mean of beta, (β_2) which is a Markov Chain-Monte Carlo object containing the parameter drawn from the posterior distribution of its mean; theta, (θ) contains the parameter drawn from the posterior distribution of the square root of theta; mean of tau (τ^2) object containing the parameter draws from the posterior distribution of its mean; mean of xi, (ξ^2) object containing the parameter draws from the posterior distribution of its mean; mean of kappa, (κ^2) object containing the parameter draws from the posterior distribution of its mean; mean of lambda, (λ^2) object containing the parameter draws from the posterior distribution of its mean; mean of sigma (σ^2), an object containing the parameter draws from the posterior distribution of its mean. C0 object containing the parameter draws from the posterior distribution of C0. Other parameters specific for Ridge prior: a xi, an object containing the parameter draws from the posterior distribution of a xi; a tau, an object containing the parameter draws from the posterior distribution of a tau; lambda2 B, an object containing the parameter draws from the posterior distribution of lambda2 B; kappa2 B, an object containing the parameter draws from the posterior distribution of kappa2 B. Parameters specific for the Triple Gamma prior: mean of lambda, an object containing the parameter draws from the posterior distribution of its mean; mean of kappa, an object containing the parameter draws from the posterior distribution of its mean. Parameters specific for the Through Stochastic Volatility prior: μ, an object containing the parameter draws from the posterior distribution of the mu parameter for the stochastic volatility model on the errors; ϕ, an object containing the parameter draws from the posterior distribution of the parameter for the stochastic volatility model on the errors; σ^2, an object containing the parameter draws from the posterior distribution of the mean of σ parameter for the stochastic volatility model on the errors. For further information regarding the parameters employed and the general implementation of the model, see Knaus P, Bitto-Nemling A, Cadonna A, and Frühwirth-Schnatter S (2022). shrinkTVP: Efficient Bayesian Inference for Time-Varying Parameter Models with Shrinkage. R package version 2.05, https://CRAN.R-project.org/package=shrinkTVP.

TABLE 3. Hierarchical Bayesian Lasso-ridge Prior "Consumer" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	-0.078	0.134	-0.085	-0.322	0.227	1018
β_2 Mkt.RF	1.001	0.104	1.009	0.798	1.208	126
β_2 SMB	0.041	0.121	0.049	-0.200	0.263	195
β_2 HML	0.061	0.202	0.064	-0.366	0.438	251
β_2 RMW	0.410	0.126	0.412	0.169	0.682	352
β_2 CMA	-0.046	0.196	-0.034	-0.443	0.329	319
$\lvert\theta\rvert$ Intercept	0.015	0.013	0.012	0	0.041	780
$\lvert\theta\rvert$ Mkt.RF	0.015	0.006	0.014	0.004	0.027	277
$\lvert\theta\rvert$ SMB	0.017	0.008	0.016	0.004	0.033	424
$\lvert\theta\rvert$ HML	0.056	0.014	0.055	0.031	0.086	209
$\lvert\theta\rvert$ RMW	0.013	0.010	0.011	0	0.032	373
$\lvert\theta\rvert$ CMA	0.043	0.020	0.041	0.002	0.079	181
σ^2	1.802	0.120	1.799	1.576	2.037	934
C_0	1.942	0.699	1.859	0.663	3.307	5000

Notes: The specifications are similar to those in Table 2.

dynamic may not have wide fluctuations, the same cannot be said for the former, which can be heavily impacted by different endogenous and exogenous factors.

In a similar manner to the previous Hierarchical Bayesian Lasso, the Mkt.RF variable exhibits the same evolution concerning periods of economic downturn. As can be observed in the figure, the troughs can be identified during the stagflation period of the 1980s, during the collapse of the Dotcom bubble, and, lastly, during the GFC. A similar evolution can be observed in the case of the profitability factor, RMW, which showcases a similar evolution in line with the previous model.

A surprise appears in the form of the "value" factor, namely HML. In this case, one can note that during economic downturns the time variable effects of this factor exhibit certain spikes. To this end, we may argue that investors and market participants in general prefer to reduce risk and uncertainty regarding their portfolio composition. We argue that a certain preference appears for the so-called "value" companies which generally pay dividends over the so-called "growth" companies that usually do not pay dividends but perform buybacks. As can be seen in Figure 4, spikes appear during economic downturns such as the 1970s and 1980s or the end of the Dotcom bubble. While on the other hand, troughs can be seen during periods of high economic growth such as the period that stretches from the late 1980s to the late 1990s. It is also interesting to note, the fact that during the last decade spanning from the 2010s up to today, this factor tends to redundancy, as can be showcased by the trough that occurred during the pandemic crisis.

Nevertheless, as the graphical representations in Figure 3 and Figure 4 show, all the parameters present a certain degree of variability over time. The next model of interest is represented by the Hierarchical Bayesian Lasso Double Gamma prior, which is the first model that has more than 1 prior. Given this, the results obtained for this model should provide a clearer image of the time variability of the factors employed. The results of this analysis can be viewed in Table 4.

This model confirms the results presented in the previous 2 models, namely, the fact that yet again, the highest time variability is showcased by the Mkt.RF variable. This time, the beta of the Mkt.RF variable is 1.065 followed by the RMW variable with a beta of 0.376 while the other variables seem to express a relatively weak variability over time. This can be

TABLE 4. Hierarchical Bayesian Lasso-Double Gamma Prior "Consumer" Industry

Parameter	Mean	SD	Median	HPD 2.5%	HPD 97.5%	ESS
β_2 Intercept	-0.032	0.062	-0.003	-0.176	0.048	841
β_2 Mkt.RF	1.065	0.084	1.065	0.900	1.241	143
β_2 SMB	0.015	0.059	0.000	-0.085	0.163	251
β_2 HML	0.018	0.107	0.000	-0.224	0.261	238
β_2 RMW	0.376	0.103	0.365	0.195	0.620	340
β_2 CMA	0.003	0.091	0.000	-0.174	0.246	245
$\|\theta\|$ Intercept	0.005	0.008	0.001	0.000	0.021	409
$\|\theta\|$ Mkt.RF	0.012	0.005	0.012	0.003	0.022	347
$\|\theta\|$ SMB	0.014	0.007	0.014	0.000	0.027	260
$\|\theta\|$ HML	0.056	0.015	0.054	0.028	0.085	213
$\|\theta\|$ RMW	0.005	0.007	0.003	0.000	0.020	189
$\|\theta\|$ CMA	0.028	0.017	0.025	0.000	0.059	122
τ^2 Intercept	0.420	6.677	0.001	0.000	0.469	4010
τ^2 Mkt.RF	35.453	1547	1.158	0.071	21.527	5000
τ^2 SMB	0.324	5.306	0.000	0.000	0.308	4639
τ^2 HML	0.459	6.620	0.000	0.000	0.623	5000
τ^2 RMW	3.041	28.372	0.244	0.000	7.732	4403
τ^2 CMA	0.660	15.620	0.000	0.000	0.522	5000
ξ^2 Intercept	0.005	0.141	0.000	0.000	0.006	5000
ξ^2 Mkt.RF	0.014	0.195	0.000	0.000	0.023	3730
ξ^2 SMB	0.011	0.155	0.001	0.000	0.024	2905
ξ^2 HML	0.043	0.531	0.005	0.000	0.112	4769
ξ^2 RMW	0.007	0.297	0.000	0.000	0.008	5000
ξ^2 CMA	0.040	0.701	0.002	0.000	0.054	4326
ξ	0.125	0.045	0.119	0.048	0.218	428
τ	0.094	0.037	0.089	0.029	0.164	213
$\kappa^2 B$	322.882	486.750	146.734	0.000	1224.981	3414
$\lambda^2 B$	8.106	19.028	1.813	0.000	37.937	2887
σ^2	1.861	0.119	1.861	1.624	2.088	1075
C_0	1.930	0.719	1.833	0.705	3.372	5000

Notes: The specifications are similar to those in Table 2.

confirmed by the posterior density distributions for the variables. The fact that this time, the RMW variable or the Profitability factor expresses the second most time-varying score, signals the fact that besides the dynamic of the stock market, the profitability of the companies can also have certain fluctuations. Namely, the profit of companies may expand during economic expansions or credit booms, while on the other hand, during recessions or inflationary periods, their profits may diminish.

Another interesting aspect may be found in the present environment, namely, the fact that companies are faced with inflationary pressure in both energy and materials sectors while the consumers themselves, struggling under the same inflationary pressure, tend to reduce consumption. This can also be confirmed by the fact that, as one may note in Figure 6, RMW has a downward trend. The cost of materials and energy has by far the largest impact on this factor, as can be seen from its evolution during the 2020s. While in the case of Mkt.RF, the explanations remain unchanged. The graphical representations of those results can be viewed in Figure 5 and Figure 6. The next model in question is represented by the Bayesian Lasso-Triple Gamma prior, which employs 3 priors to better capture the time-varying effects of different variables. The results obtained can be viewed in Table 5.

Similarly, to the previous Double Gamma model, with 2 priors, this model confirms the findings and seems to be in line with all the previous models as well. The results suggest that yet again the Mkt.RF variable with a beta of 1.078 presents the largest time variability amongst its peers. Only to be followed by the RMW variable with a beta of 0.393, which can confirm the fact that besides the stock market, the profitability of companies can witness fluctuations. For the other parameters, it can be noted that they exhibit a certain time variability, albeit low and, they can be considered rather insignificant. It can also be argued, as in the case of the previous models, that the evolution of the Mkt.RF variable exhibits the same trend and troughs. It can be easily linked to the evolution of the market and its current phase, be it a "Bear" or a "Bull" market.

Another interesting evolution can be seen in the case of the profitability factor, RMW which, as previously explained has a downward trend. This evolution, as in the previous cases, and taking into account the specifics of the industry, can be attributed to rising costs for both energy and materials. Lastly, regarding the investment factor, CMA although presenting a relatively low time-varying effect with a beta of 0.013, the graphical representation from Figure 8 can present certain valuable insights. As can be noted, the peaks occur during periods of economic growth, such as the late 1980s up to the early 1990s period. Another interesting aspect can be viewed starting from 2000, when, it can be argued that the process of globalization and the demand for goods and services from both the internal and the external markets increased drastically.

As a consequence, from the 2000s up to the 2020s, the "Consumer" Industry started to invest more aggressively to increase production capacity. The graphical representation of the results can be viewed in Figure 7 and Figure 8. Finally, the last model that was performed on the "Consumer" Industry, Through Stochastic Volatility, confirms the previously reported results of the earlier models employed. The results of this analysis can be viewed in Table 6.

As was the case in the previous models, the strongest variable, Mkt.RF with a beta of 1.096, the largest beta value out of all the previous models confirms the previous models. An interesting aspect is the fact that the RMW variable has a lower beta value of just 0.351 while during the former model with 3 priors, it had a beta value of 0.393. This can be explained by the fact that generally speaking the stochastic methods are better suited for returns rather than scores. Similarly, this can explain why the value of the Mkt.RF variable has the highest value in this model. The graphical representation of the results can be viewed in Figure 9 and Figure 10. It can be argued, as in the case of the previous models that the evolution of the most significant variables is influenced by the same factors. It is also important to note the fact that this final model, attaches large importance to the excess return of the market factor, Mkt. RF. While the remaining factors tend to be overshadowed by this preference. Lastly, it can be argued that in the case of the "Consumer" Industry, the excess return of the market, the profitability, and a lesser degree of significance, the investment factor, tend to capture the price formation mechanism faithfully.

5.2. **Results for the "Manufacturing" Industry.** The next industry analyzed in this paper is the "Manufacturing" Industry. Similarly, in the "Consumer" Industry, the models involved are the same.

Similarly, in the "Consumer" Industry, the most statistically significant factors are the Mkt.RF, and RMW. Nevertheless, as one may note, the HML and the CMA factors are also statistically significant. We argue that the results obtained are partially in line with the results obtained from the shrinkTVP framework with a few exceptions. Namely, the most relevant factors for both models are indeed the Mkt.RF and the RMW factors, while the GLM assigns

Table 5. Hierarchical Bayesian Lasso-Triple Gamma Prior “Consumer” Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	-0.035	0.060	-0.006	-0.179	0.039	775
β_2 Mkt.RF	1.078	0.086	1.076	0.909	1.256	151
β_2 SMB	0.029	0.066	0.000	-0.069	0.194	256
β_2 HML	0.009	0.106	0.000	-0.225	0.272	218
β_2 RMW	0.393	0.117	0.377	0.185	0.677	313
β_2 CMA	0.013	0.102	0.000	-0.177	0.250	247
$\|\theta\|$ Intercept	0.004	0.008	0.001	0.000	0.019	505
$\|\theta\|$ Mkt.RF	0.013	0.005	0.012	0.003	0.023	301
$\|\theta\|$ SMB	0.014	0.008	0.014	0.000	0.028	228
$\|\theta\|$ HML	0.056	0.014	0.054	0.031	0.084	225
$\|\theta\|$ RMW	0.007	0.008	0.004	0.000	0.023	251
$\|\theta\|$ CMA	0.031	0.018	0.030	0.000	0.064	136
τ^2 Intercept	0.036	0.148	0.000	0.000	0.175	1075
τ^2 Mkt.RF	0.314	0.525	0.103	0.000	1.323	1891
τ^2 SMB	0.047	0.211	0.000	0.000	0.222	1398
τ^2 HML	0.042	0.181	0.000	0.000	0.220	2096
τ^2 RMW	0.197	0.406	0.043	0.000	0.923	1322
τ^2 CMA	0.044	0.193	0.000	0.000	0.239	2405
ξ^2 Intercept	0.037	0.187	0.000	0.000	0.169	2842
ξ^2 Mkt.RF	0.087	0.263	0.006	0.000	0.468	1862
ξ^2 SMB	0.093	0.259	0.006	0.000	0.503	1595
ξ^2 HML	0.208	0.421	0.044	0.000	1.003	1183
ξ^2 RMW	0.051	0.192	0.001	0.000	0.271	2017
ξ^2 CMA	0.149	0.366	0.018	0.000	0.742	1277
λ^2 Intercept	0.486	0.695	0.218	0.000	1.891	2757
λ^2 Mkt.RF	0.221	0.428	0.049	0.000	1.035	1112
λ^2 SMB	0.476	0.673	0.211	0.000	1.834	2770
λ^2 HML	0.460	0.681	0.188	0.000	1.774	2578
λ^2 RMW	0.327	0.543	0.106	0.000	1.385	1486
λ^2 CMA	0.456	0.676	0.192	0.000	1.809	2675
κ^2 Intercept	0.495	0.714	0.222	0.000	1.935	2636
κ^2 Mkt.RF	0.438	0.643	0.191	0.000	1.702	2677
κ^2 SMB	0.444	0.645	0.184	0.000	1.724	2573
κ^2 HML	0.309	0.518	0.102	0.000	1.336	2079
κ^2 RMW	0.473	0.677	0.199	0.000	1.781	2943
κ^2 CMA	0.389	0.597	0.141	0.000	1.605	2848
a_ξ	0.161	0.052	0.154	0.060	0.259	565
c_ξ	0.378	0.071	0.389	0.240	0.496	760
a_τ	0.133	0.057	0.129	0.027	0.235	80
c_τ	0.383	0.068	0.393	0.246	0.490	872
$\kappa^2 B$	6712.160	59875.300	7361.052	0.016	12712.589	404
$\lambda^2 B$	233.892	2178.590	7.611	0.000	374.853	340
σ^2	1.854	0.123	1.850	1.619	2.095	706
C_0	1.929	0.718	1.833	0.659	3.352	5000

Notes: The specifications are similar to those in Table 2.

TABLE 6. Through Stochastic Volatility, "Consumer" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	-0.017	0.051	0.000	-0.151	0.049	609
β_2 Mkt.RF	1.096	0.074	1.092	0.961	1.252	206
β_2 SMB	0.009	0.050	0.000	-0.072	0.143	172
β_2 HML	0.003	0.076	0.000	-0.194	0.178	313
β_2 RMW	0.351	0.089	0.351	0.211	0.566	147
β_2 CMA	0.002	0.090	0.000	-0.203	0.221	272
$\lvert\theta\rvert$ Intercept	0.007	0.011	0.001	0.000	0.033	165
$\lvert\theta\rvert$ Mkt.RF	0.010	0.004	0.010	0.003	0.019	287
$\lvert\theta\rvert$ SMB	0.015	0.007	0.014	0.000	0.028	208
$\lvert\theta\rvert$ HML	0.043	0.014	0.041	0.017	0.069	85
$\lvert\theta\rvert$ RMW	0.003	0.006	0.001	0.000	0.016	126
$\lvert\theta\rvert$ CMA	0.032	0.014	0.031	0.008	0.062	130
τ^2 Intercept	0.413	16.786	0.000	0.000	0.207	5000
τ^2 Mkt.RF	40.307	1744.300	1.258	0.062	25.781	5000
τ^2 SMB	0.945	55.684	0.000	0.000	0.163	5000
τ^2 HML	0.253	3.404	0.000	0.000	0.406	5000
τ^2 RMW	10.127	321.070	0.218	0.000	7.364	2671
τ^2 CMA	0.934	49.836	0.000	0.000	0.336	5000
ξ^2 Intercept	0.004	0.037	0.000	0.000	0.009	2657
ξ^2 Mkt.RF	0.568	39.621	0.000	0.000	0.018	5000
ξ^2 SMB	0.022	0.741	0.001	0.000	0.027	5000
ξ^2 HML	0.033	0.318	0.003	0.000	0.084	4554
ξ^2 RMW	0.011	0.490	0.000	0.000	0.003	5000
ξ^2 CMA	0.035	0.442	0.002	0.000	0.060	4316
ξ	0.123	0.046	0.116	0.043	0.212	319
τ	0.090	0.038	0.083	0.025	0.163	164
$\kappa^2 B$	332.230	484.910	152.080	0.000	1262.218	3428
$\lambda^2 B$	7.905	17.792	1.756	0.000	36.018	3411
μ	0.464	0.115	0.464	0.243	0.695	1114
ϕ	0.853	0.080	0.869	0.701	0.970	50
σ^2	0.101	0.055	0.091	0.018	0.210	63

Notes: The specifications are similar to those in Table 2.

TABLE 7. Results for the Generalized Linear Model for the "Manufacturing" Industry (MNF)

Parameter	**Estimate (Standard Error)**
Intercept	-0.155 (0.065)
Mkt.RF	0.970*** (0.017)
SMB	-0.003 (0.024)
HML	0.142*** (0.040)
RMW	0.249*** (0.036)
CMA	0.220*** (0.051)

Notes: Standard errors are reported in parentheses. *, **, *** indicates significance at the 90%, 95%, and 99% levels, respectively. The specifications are similar to those in Table 1.

larger importance to factors such as HML, CMA, and SMB due to the nature of the model involved. Nevertheless, as can be seen in the results of the shrinkTVP framework, the HML and the CMA showcase the weakest time variability. Hence, the first model in question is the Hierarchical Bayesian Lasso, the results of which can be found in Table 8.

TABLE 8. Hierarchical Bayesian Lasso for "Manufacturing" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	-0.082	0.117	-0.082	-0.335	0.150	1246
β_2 Mkt.RF	1.070	0.094	1.069	0.888	1.261	149
β_2 SMB	0.058	0.106	0.061	-0.168	0.249	194
β_2 HML	0.016	0.182	0.015	-0.347	0.378	277
β_2 RMW	0.413	0.132	0.414	0.111	0.659	323
β_2 CMA	0.011	0.167	0.015	-0.333	0.324	331
$\|\theta\|$ Intercept	0.011	0.010	0.008	0	0.032	830
$\|\theta\|$ Mkt.RF	0.013	0.005	0.013	0.004	0.024	349
$\|\theta\|$ SMB	0.015	0.008	0.015	0.002	0.030	378
$\|\theta\|$ HML	0.053	0.014	0.053	0.028	0.082	240
$\|\theta\|$ RMW	0.012	0.009	0.010	0	0.028	287
$\|\theta\|$ CMA	0.031	0.016	0.029	0.003	0.064	225
τ^2 Intercept	0.255	0.695	0.092	0	0.919	1206
τ^2 Mkt.RF	0.616	0.710	0.414	0.076	1.750	1157
τ^2 SMB	0.240	0.574	0.086	0	0.928	1222
τ^2 HML	0.273	0.701	0.099	0	1.014	1276
τ^2 RMW	0.374	0.703	0.197	0.001	1.236	1324
τ^2 CMA	0.272	0.799	0.098	0	0.982	1874
ξ^2 Intercept	0.002	0.004	0.001	0	0.009	1816
ξ^2 Mkt.RF	0.002	0.004	0.001	0	0.008	2020
ξ^2 SMB	0.002	0.005	0.001	0	0.009	2207
ξ^2 HML	0.004	0.005	0.002	0	0.012	1134
ξ^2 RMW	0.002	0.004	0.001	0	0.009	1728
ξ^2 CMA	0.003	0.004	0.002	0	0.010	1710
κ^2 B	963.545	698.950	790.720	64.816	2352.265	979
λ^2 B	11.159	10.101	8.330	0.161	30.165	819
σ^2	1.840	0.117	1.836	1.618	2.069	998
C_0	1.924	0.705	1.840	0.657	3.256	5000

Notes: The specifications are similar to those in Table 2.

In this model, the variables that exhibit the highest time variability are the Mkt.RF variable with a beta value of 1.07 and the RMW variable with a beta value of 0.413. These variables are followed by the SMB variable with a beta value of just 0.058, while the other variables do show a certain degree of time variability, albeit lower. An explanation for those results may be found in the sector of these publicly traded companies. Namely, the fact that this industry serves to provide different types of equipment and durable goods to the economy. Due to this, it can be argued that demand for such goods may be impacted by the state of the economy. During periods of economic growth and expansion, both the profitability and the returns provided by such companies can witness a significant increase. While, on the other hand, during economic downturns, profitability and returns may be negatively affected. It can also be reasonable stated that, compared to the "Consumer" Industry, the profitability factor exhibits larger time variations.

In a similar manner to the "Consumer" Industry, it can be noted that for both variables, Mkt.RF and RMW, the evolution and explanations behind this evolution are similar. A distinction should be drawn nevertheless, to this end, we argue that both the "Consumer" and "Manufacturing" Industries exhibit a large degree of sensitivity to the general economic conditions. As a consequence, their evolutions can be pinpointed as favorable during economic booms and as unfavorable during economic downturns or recessions. The graphical representations of those results can be viewed in Figure 11 and Figure 12. The next model employed is the Hierarchical Bayesian Lasso-ridge prior, which uses a single prior. The results obtained from this model can be viewed in Table 9.

TABLE 9. Hierarchical Bayesian Lasso-Ridge Prior for "Manufacturing" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	-0.078	0.134	-0.085	-0.322	0.227	1018
β_2 Mkt.RF	1.001	0.104	1.009	0.798	1.208	126
β_2 SMB	0.041	0.121	0.049	-0.200	0.263	195
β_2 HML	0.061	0.202	0.064	-0.366	0.438	251
β_2 RMW	0.410	0.126	0.412	0.169	0.682	352
β_2 CMA	-0.046	0.196	-0.034	-0.443	0.329	319
$\lvert\theta\rvert$ Intercept	0.015	0.013	0.012	0	0.041	780
$\lvert\theta\rvert$ Mkt.RF	0.015	0.006	0.014	0.004	0.027	277
$\lvert\theta\rvert$ SMB	0.017	0.008	0.016	0.004	0.033	424
$\lvert\theta\rvert$ HML	0.056	0.014	0.055	0.031	0.086	209
$\lvert\theta\rvert$ RMW	0.013	0.010	0.011	0	0.032	373
$\lvert\theta\rvert$ CMA	0.043	0.020	0.041	0.002	0.079	181
σ^2	1.802	0.120	1.799	1.576	2.037	934
C_0	1.942	0.699	1.859	0.663	3.307	5000

Notes: The specifications are similar to those in Table 2.

The results obtained in this model are in line with the previous Bayesian Lasso analyses, namely the fact that the main variables of interest continue to be the Mkt.RF and RMW variables. In the case of the ridge prior, the beta value of the Mkt.RF variable is 1.001, lower than in the previous model, while the RMW variable has a beta value of just 0.41. A difference can be observed in the case of the SMB variable, which suffered a significant decrease from the different model to a beta value of just 0.041, while the HML variable seems to have increased to a beta of 0.061. Similarly, the beta of the CMA variable has a larger value of 0.046, albeit negative.

The same explanation as in the case of the previous models can be put forth in the case of the Mkt.RF and RMW variables. While in the case of the HML factor, it can be argued that investors opted to diversify away from growth companies during times of economic distress, such as the 1980s and 2000 or 2008. Thus, this can confirm that investors search for a way to safeguard their capital during economic distress. An interesting observation in the case of this industry can be drawn from the SMB variable or the size factor. Given its construction, we argue that investors have preferred smaller companies in the years spanning from the late 1970s up to the early 2000s. An explanation can be attributed to the fact that investors actively preferred companies that present a growth potential or companies that can revolutionize certain elements that are in line with the "Manufacturing" Industry.

To this end, smaller but revolutionary "Manufacturing" companies could have been preferred over the large and already established ones with declining growth possibilities. Another element that is important to note is also the entry barrier, which is significant, especially in the case of retail investors. Thus, we argue that the prospect of possible growth, coupled with a lesser entry barrier could have been decisive factors. Another aspect worth noting may revolve around the financing and financial health of larger "Manufacturing" companies. To this end, we consider that investors would avoid over-indebted companies. The graphical representations can be viewed in Figure 13 and Figure 14. The next model applied in the case of the "Manufacturing" Industry is the Hierarchical Bayesian Lasso-Double Gamma prior, which uses 2 priors. The results obtained from this model can be viewed in Table 10.

Table 10. Hierarchical Bayesian Lasso-Double Gamma Prior for "Manufacturing" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	-0.032	0.062	-0.003	-0.176	0.048	841
β_2 Mkt.RF	1.065	0.084	1.065	0.900	1.241	143
β_2 SMB	0.015	0.059	0	-0.085	0.163	251
β_2 HML	0.018	0.107	0	-0.224	0.261	238
β_2 RMW	0.376	0.103	0.365	0.195	0.620	340
β_2 CMA	0.003	0.091	0	-0.174	0.246	245
$\|\theta\|$ Intercept	0.005	0.008	0.001	0	0.021	409
$\|\theta\|$ SMB	0.014	0.007	0.014	0	0.027	260
$\|\theta\|$ HML	0.056	0.015	0.054	0.028	0.085	213
$\|\theta\|$ RMW	0.005	0.007	0.003	0	0.020	189
$\|\theta\|$ CMA	0.028	0.017	0.025	0	0.059	122
τ^2 Intercept	0.420	6.677	0.001	0	0.469	4010
τ^2 Mkt.RF	35.453	1547	1.158	0.071	21.527	5000
τ^2 SMB	0.324	5.306	0	0	0.308	4639
τ^2 HML	0.459	6.620	0	0	0.623	5000
τ^2 RMW	3.041	28.372	0.244	0	7.732	4403
ξ^2 Intercept	0.005	0.141	0	0	0.006	5000
ξ^2 Mkt.RF	0.014	0.195	0	0	0.023	3730
ξ^2 SMB	0.011	0.155	0.001	0	0.024	2905
ξ^2 HML	0.043	0.531	0.005	0	0.112	4769
ξ^2 RMW	0.007	0.297	0	0	0.008	5000
λ^2 B	8.106	19.028	1.813	0	37.937	2887
σ^2	1.861	0.119	1.861	1.624	2.088	1075
C_0	1.930	0.719	1.833	0.705	3.372	5000

Notes: The specifications are similar to those in Table 2.

The results obtained are in line with the previous models, namely that the variables Mkt.RF and RMW exhibit the largest time variability. The beta of RMW has a score of 0.376 while the Mkt.RF variable has a beta score of 1.065. The other variables exhibit a lower degree of time variability which can be confirmed by the highest posterior distribution. Thus, similarly, the arguments presented above, are confirmed by the results provided in this model. The next model employed in the case of the "Manufacturing" Industry is represented by the Hierarchical

Bayesian Lasso-Triple Gamma prior, which employs a number of 3 priors to accurately detect time-varying variables and avoid overfitting. The results can be viewed in Table 11.

TABLE 11. Hierarchical Bayesian Lasso-Triple Gamma Prior for "Manufacturing" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	-0.035	0.060	-0.006	-0.179	0.039	775
β_2 Mkt.RF	1.078	0.086	1.076	0.909	1.256	151
β_2 SMB	0.029	0.066	0	-0.069	0.194	256
β_2 HML	0.009	0.106	0	-0.225	0.272	218
β_2 RMW	0.393	0.117	0.377	0.185	0.677	313
β_2 CMA	0.013	0.102	0	-0.177	0.250	247
$\lvert\theta\rvert$ Intercept	0.004	0.008	0.001	0	0.019	505
$\lvert\theta\rvert$ SMB	0.014	0.008	0.014	0	0.028	228
$\lvert\theta\rvert$ HML	0.056	0.014	0.054	0.031	0.084	225
$\lvert\theta\rvert$ RMW	0.007	0.008	0.004	0	0.023	251
$\lvert\theta\rvert$ CMA	0.031	0.018	0.030	0	0.064	136
τ^2 Intercept	0.036	0.148	0	0	0.175	1075
τ^2 Mkt.RF	0.314	0.525	0.103	0	1.323	1891
τ^2 SMB	0.047	0.211	0	0	0.222	1398
τ^2 HML	0.042	0.181	0	0	0.220	2096
τ^2 RMW	0.197	0.406	0.043	0	0.923	1322
ξ^2 Intercept	0.037	0.187	0	0	0.169	2842
ξ^2 Mkt.RF	0.087	0.263	0.006	0	0.468	1862
ξ^2 SMB	0.093	0.259	0.006	0	0.503	1595
ξ^2 HML	0.208	0.421	0.044	0	1.003	1183
ξ^2 RMW	0.051	0.192	0.001	0	0.271	2017
κ^2 Mkt.RF	0.438	0.643	0.191	0	1.702	2677
κ^2 HML	0.309	0.518	0.102	0	1.336	2079
κ^2 RMW	0.473	0.677	0.199	0	1.781	2943
κ^2 CMA	0.389	0.597	0.141	0	1.605	2848
ξ	0.161	0.052	0.154	0.060	0.259	565
$c\,\xi$	0.378	0.071	0.389	0.240	0.496	760
τ	0.133	0.057	0.129	0.027	0.235	80
$c\,\tau$	0.383	0.068	0.393	0.246	0.490	872
κ^2 B	6712.200	59875	361.052	0.016	12712.580	404
λ^2 B	233.890	2178.600	7.611	0	374.853	340
σ^2	1.854	0.123	1.850	1.619	2.095	706
C_0	1.929	0.718	1.833	0.659	3.352	5000

Notes: The specifications are similar to those in Table 2.

Following the trend of the previous models, the variables that show the highest time variability are the Mkt.RF variable, with a beta value of 1.078 followed by the RMW variable with a beta value of 0.393. It is also worthwhile to note that the remainder variables are starting to exhibit larger values and, implicitly, are varying in time. As in the case of both the "Consumer" and "Manufacturing" Industries, these variables tend to be the most important. In the case of RMW, the explanation noted above remains coherent. In the case of SMB and HML, it can be noted that although the time-varying effects are lower, the general economic explanations tend to remain valid. Lastly, the Through Stochastic Volatility model was performed in the "Manufacturing" Industry. The results obtained can be viewed in Table 12.

Table 12. Through Stochastic Volatility, "Manufacturing" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 SMB	0.009	0.050	0	-0.072	0.143	172
β_2 HML	0.003	0.076	0	-0.194	0.178	313
β_2 RMW	0.351	0.089	0.351	0.211	0.566	147
β_2 CMA	0.002	0.090	0	-0.203	0.221	272
$\lvert\theta\rvert$ Intercept	0.007	0.011	0.001	0	0.033	165
$\lvert\theta\rvert$ Mkt.RF	0.010	0.004	0.010	0.003	0.019	287
$\lvert\theta\rvert$ SMB	0.015	0.007	0.014	0	0.028	208
$\lvert\theta\rvert$ HML	0.043	0.014	0.041	0.017	0.069	85
$\lvert\theta\rvert$ RMW	0.003	0.006	0.001	0	0.016	126
$\lvert\theta\rvert$ CMA	0.032	0.014	0.031	0.008	0.062	130
τ^2 Intercept	0.413	16.790	0	0	0.207	5000
τ^2 Mkt.RF	40.307	1744	1.258	0.062	25.781	5000
τ^2 SMB	0.945	55.680	0	0	0.163	5000
τ^2 HML	0.253	3.404	0	0	0.406	5000
τ^2 RMW	10.127	321.100	0.218	0	7.364	2671
τ^2 CMA	0.934	49.840	0	0	0.336	5000
ξ^2 Intercept	0.004	0.037	0	0	0.009	2657
ξ^2 Mkt.RF	0.568	39.620	0	0	0.018	5000
ξ^2 SMB	0.022	0.741	0.001	0	0.027	5000
ξ^2 HML	0.033	0.318	0.003	0	0.084	4554
ξ^2 RMW	0.011	0.490	0	0	0.003	5000
ξ^2 CMA	0.035	0.442	0.002	0	0.060	4316
ξ	0.123	0.046	0.116	0.043	0.212	319
τ	0.090	0.038	0.083	0.025	0.163	164
κ^2 B	332.230	484.900	152.080	0	1262.218	3428
λ^2 B	7.905	17.790	1.756	0	36.018	3411
μ	0.464	0.115	0.464	0.243	0.695	1114
ϕ	0.853	0.080	0.869	0.701	0.970	50
σ^2	0.101	0.055	0.091	0.018	0.210	63

Notes: The specifications are similar to those in Table 2.

As in the case of the Stochastic Volatility applied to the "Consumer" Industry, one may observe certain aspects. Namely, the value of the market variable Mkt.RF has the highest value of all the previously applied models, with a beta value of 1.096. Following the same path, the second variable with the highest beta score is RMW, with a score of 0.351. Albeit with a lower value than in the case of the Triple Gamma model, we can still consider that the Stochastic model has a preference for market returns.

Given this, it can be argued that due to the industry specifics both the "Consumer" and the "Manufacturing" Industries tend to exhibit the same reactions and sensitivity to the general economic outlook. With these considerations in mind, we argue that the excess return factor, Mkt.RF and the profitability factor, RMW are the leading factors identified in the price formation mechanism for this industry. It can also be noted that both industries may have a similar evolution and reaction when faced with the effects of both exogenous and endogenous shocks.

5.3. **Results for the "Technology" Industry.** As in the case of the previous industries, the same methodology is applied to the "Technology Industry", starting with the GLM model, the results of which can be seen in Table 13.

Table 13. Results for the Generalized Linear Model for the "Technology" Industry

Parameter	**Estimate (SE)**
Intercept	0.332*** (0.079)
Mkt.RF	0.984*** (0.023)
SMB	-0.059*** (0.034)
HML	-0.208*** (0.040)
RMW	-0.405*** (0.048)
CMA	-0.292*** (0.068)

*Standard errors are reported in parentheses. *, **, *** indicates significance at the 90%, 95%, and 99% level, respectively.*
Notes: The specifications are similar to those in Table 1.

The results obtained within the GLM framework suggest that the most relevant factor in the case of the "Technology" factors is the Mkt.RF and RMW factors, followed by the SMB factor. It can be argued that the results from the GLM are in line with the results provided by the shrinkTVP model, namely: The most relevant factors are the Mkt.RF, the RMW factor, and, lastly, the SMB factor. We argue that the GLM assigns a larger statistical probability to the CMA factor due to its static parameters employed. Nevertheless, we consider that the results from both models are relevant in presenting the relevance of the Mkt.RF, RMW, and SMB factors within the "Technology" Sector. Therefore, the starting model for the shrinkTVP model is represented by the Hierarchical Bayesian Lasso, the results can be viewed in Table 14.

The results obtained seem to be in line with the previous industries, namely, the Mkt.RF and RMW variables exhibit the largest time variability of the set. The beta value of Mkt.RF is 1.07 while the beta value of RMW is 0.413. It can also be noted the fact that the SMB or the Size factor starts to play a role in this industry. The beta value of SMB is 0.058, which can be interpreted as the staple of this industry. In other words, the majority of companies that are involved in the "Technology" Industry are, in their majority, large in size at present. It can also be argued that the observation window which started in 1963, may exert a certain impact on this factor and, on the industry as a whole. The results come as no surprise, especially given the nature of the excess market return factor or the profitability factor.

A surprise nevertheless can be seen in the case of the size factor, SMB. It can be seen that for this particular industry, starting from the mid-1970s, small-sized companies have been preferred. This can be attributed mainly to the fact that in general, small-size firms with a revolutionary product or idea are the main drivers in this industry. Nevertheless, as the graphical representations show, the companies that had the greatest impact were, at the time of writing, of large size. Given this, starting from the early 2000s, and after the collapse of the Dotcom bubble, investors preferred to allocate capital to large and well-established tech companies.

This trend only exacerbates in the period post-GFC, as can be seen in the graph. Starting from the 2010s, larger size tech companies such as Facebook, Microsoft, and Google, or the famous FANG selection dominated the portfolio allocation preferences of investors. The graphical representations of those results can be viewed in Figure 20 and Figure 21. The next model employed is Hierarchical Bayesian Lasso-ridge prior, which makes use of a ridge prior to better

TABLE 14. Hierarchical Bayesian Lasso for "Technology" Industry

Parameter	Mean	SD	Median	HPD 2.5%	HPD 97.5%	ESS
β_2 Intercept	-0.082	0.117	-0.082	-0.335	0.150	1246
β_2 Mkt.RF	1.070	0.094	1.069	0.888	1.261	149
β_2 SMB	0.058	0.106	0.061	-0.168	0.249	194
β_2 HML	0.016	0.182	0.015	-0.347	0.378	277
β_2 RMW	0.413	0.132	0.414	0.111	0.659	323
β_2 CMA	0.011	0.167	0.015	-0.333	0.324	331
$\|\theta\|$ Intercept	0.011	0.010	0.008	0	0.032	830
$\|\theta\|$ SMB	0.015	0.008	0.015	0.002	0.030	378
$\|\theta\|$ HML	0.053	0.014	0.053	0.028	0.082	240
$\|\theta\|$ RMW	0.012	0.009	0.010	0	0.028	287
$\|\theta\|$ CMA	0.031	0.016	0.029	0.003	0.064	225
τ^2 Intercept	0.255	0.695	0.092	0	0.919	1206
τ^2 Mkt.RF	0.616	0.710	0.414	0.076	1.750	1157
τ^2 SMB	0.240	0.574	0.086	0	0.928	1222
τ^2 HML	0.273	0.701	0.099	0	1.014	1276
τ^2 RMW	0.374	0.703	0.197	0.001	1.236	1324
τ^2 CMA	0.272	0.799	0.098	0	0.982	1874
ξ^2 Intercept	0.002	0.004	0.001	0	0.009	1816
ξ^2 Mkt.RF	0.002	0.004	0.001	0	0.008	2020
ξ^2 SMB	0.002	0.005	0.001	0	0.009	2207
ξ^2 HML	0.004	0.005	0.002	0	0.012	1134
ξ^2 RMW	0.002	0.004	0.001	0	0.009	1728
ξ^2 CMA	0.003	0.004	0.002	0	0.010	1710
$\kappa^2 B$	963.540	5698.950	3790.720	64.816	2352.265	979
$\lambda^2 B$	11.159	10.101	8.330	0.161	30.165	819
σ^2	1.840	0.117	1.836	1.618	2.069	998
C_0	1.924	0.705	1.840	0.657	3.256	5000

Notes: The specifications are similar to those in Table 2.

forecast and present the time variability of selected variables. The results of this model can be viewed in Table 15.

Following a similar trend to the previous model, one may observe that the model captured the most variations in time for the Mkt.RF variable which has a beta value of 1.001. The main factors that can be attributed to this are the market conditions. As was noted in the case of the previous industries and models, the excess return of the market is influenced by the general economic circumstances and the phase of the market. Followed by the RMW variable with a beta of 0.41 which seems to present a similar downward evolution over the analyzed period.

An interesting result is posed by the HML variable or the Value factor with a beta value of 0.061. An argument in the case of this evolution can be best explained by the Dotcom bubble collapse in 2000. During this time, investors chose to divest from aggressive growth companies from the tech sector that have been largely propped up by the general exuberance of the public towards this industry. Given this, it can be argued that as in the case of the previous industries, during times of economic distress or large financial crisis, holding "value" companies is a preferable option to holding "growth" companies. In the case of the remaining variables, it can be stated that they exhibit a certain variation over time, albeit lower.

The explanations formulated for the previous model hold in the case of this model which utilizes a ridge prior. Nevertheless, one may note the fact that the preference of investors to allocate resources to large-size companies remains unhinged. The next model applied to the

Table 15. Hierarchical Bayesian Lasso-ridge Prior "Technology" Industry

Parameter	Mean	SD	Median	HPD 2.5%	HPD 97.5%	ESS
β_2 Intercept	-0.078	0.134	-0.085	-0.322	0.227	1018
β_2 Mkt.RF	1.001	0.104	1.009	0.798	1.208	126
β_2 SMB	0.041	0.121	0.049	-0.200	0.263	195
β_2 HML	0.061	0.202	0.064	-0.366	0.438	251
β_2 RMW	0.410	0.126	0.412	0.169	0.682	352
β_2 CMA	-0.046	0.196	-0.034	-0.443	0.329	319
$\|\theta\|$ Intercept	0.015	0.013	0.012	0	0.041	780
$\|\theta\|$ SMB	0.017	0.008	0.016	0.004	0.033	424
$\|\theta\|$ HML	0.056	0.014	0.055	0.031	0.086	209
$\|\theta\|$ RMW	0.013	0.010	0.011	0	0.032	373
$\|\theta\|$ CMA	0.043	0.020	0.041	0.002	0.079	181
σ^2	1.802	0.120	1.799	1.576	2.037	934
C_0	1.942	0.699	1.859	0.663	3.307	5000

Notes: The specifications are similar to those in Table 2.

"Technology" Industry is the Hierarchical Bayesian Lasso-Double Gamma prior, which makes use of 2 priors to better estimate the time-varying parameters. The results obtained from this model are in Table 16.

Table 16. Hierarchical Bayesian Lasso-Double Gamma Prior for the "Technology" Industry

Parameter	Mean	SD	Median	HPD 2.5%	HPD 97.5%	ESS
β_2 Intercept	-0.032	0.062	-0.003	-0.176	0.048	841
β_2 Mkt.RF	1.065	0.084	1.065	0.900	1.241	143
β_2 SMB	0.015	0.059	0	-0.085	0.163	251
β_2 HML	0.018	0.107	0	-0.224	0.261	238
β_2 RMW	0.376	0.103	0.365	0.195	0.620	340
β_2 CMA	0.003	0.091	0	-0.174	0.246	245
$\|\theta\|$ Intercept	0.005	0.008	0.001	0	0.021	409
$\|\theta\|$ SMB	0.014	0.007	0.014	0	0.027	260
$\|\theta\|$ RMW	0.005	0.007	0.003	0	0.020	189
$\|\theta\|$ CMA	0.028	0.017	0.025	0	0.059	122
τ^2 Intercept	0.420	6.677	0.001	0	0.469	4010
τ^2 Mkt.RF	35.453	1547	1.158	0.071	21.527	5000
τ^2 SMB	0.324	5.306	0	0	0.308	4639
τ^2 HML	0.459	6.620	0	0	0.623	5000
τ^2 RMW	3.041	28.370	0.244	0	7.732	4403
ξ^2 Intercept	0.005	0.141	0	0	0.006	5000
ξ^2 Mkt.RF	0.014	0.195	0	0	0.023	3730
ξ^2 SMB	0.011	0.155	0.001	0	0.024	2905
ξ^2 HML	0.043	0.531	0.005	0	0.112	4769
ξ^2 RMW	0.007	0.297	0	0	0.008	5000
$\lambda^2 B$	8.106	19.03	1.813	0	37.937	2887
σ^2	1.861	0.119	1.861	1.624	2.088	1075
C_0	1.930	0.719	1.833	0.705	3.372	5000

Notes: The specifications are similar to those in Table 2.

Similarly, to the previous models, it can be observed that the most time-varying fluctuations have been observed in the case of Mkt.RF, with a beta value of 1.065, and in the case of the RMW variable with a beta value of 0.376. In the case of the other variables, it can be seen that they exhibit a certain variability over time, albeit lower. As can be noted in the graphs below, the main points discussed above hold even in the case of this model. The next model employed in the case of the "Manufacturing" Industry is represented by the Hierarchical Bayesian Lasso-Triple Gamma prior, which employs a number of 3 priors to accurately detect time-varying variables and avoid overfitting. The results obtained can be viewed in Table 17.

It can be noted that the main factors that exhibit time variable effects are the excess return of the market, Mkt.RF factor, with a beta of 1.078, and the profitability factor, RMW, with a beta of 0.393. It should be noted that as in the case of the previous models, the main drivers can be attributed to different exogenous and endogenous factors. The graphical representation of the results can be viewed in Figure 27 and Figure 28, in the Appendices section. Lastly, the last model employed, Through Stochastic Volatility is performed in the "Technology" Industry. The results obtained after applying the model can be viewed in Table 18.

As in the case of the previous industries, it can be noted that this type of model exhibits a certain preference for the excess return, namely the Mkt.RF variable, which has a beta value of 1.096. Following is the RMW variable, with a beta value of 0.351. Nevertheless, it can also be noted the fact that the other variables, SMB, HML, and CMA exhibit a certain degree of time-varying albeit, at a lower amplitude. It again can be argued that as in the case of previous Through Stochastic Volatility models, a larger significance is attributed to the excess return of the market or the Mkt.RF variable.

Given this, it may also be observed that the graphical results obtained in Figure 29 and Figure 30, are in line with the previous models. With this in mind, we argue that the economical explanations presented beforehand, namely the firm size preference of investors, coupled with a preference for "value" stocks in times of turmoil are confirmed. Although the "value" factor indicates a certain preference for this type of company, it can be argued that in the case of this industry, such companies may be difficult to identify. Given the latest development in the tech domain, at the time of writing, most companies that are held in high regard, such as Microsoft, Apple, Google, or Netflix are of the" growth" type. Thus, we argue that more research could be done in this direction.

5.4. **Results for the "Health" Industry.** The next industry of interest that we analyzed comes in the form of the "Health" Industry. Before presenting the results obtained within the shrinkTVP model, the results from the GLM model are presented.

Similarly, to the results obtained for the previous industries, we note that the most relevant factors are the Mkt.RF, CMA, HML. It is also interesting to note that, within the GLM framework, most factors showcase a negative influence on the closing price of the "Health" Industry. A similar evolution can be observed in light of the shrinkTVP model, namely, the fact that the HML and the CMA factors present the largest time variable effect. Albeit, with a negative influence on the closing price of the "Health" Industry. Given this, we argue that the GLM manages to capture the most relevant factors but fails to showcase the effect that such factors may exert on the price formation mechanism of the "Health" Industry. Following a similar approach as in the case of the previous industries, the first and foremost model from the shrinkTVP framework applied is the Hierarchical Bayesian Lasso. The results obtained can be viewed in Table 20.

As in the case of the previous industries, the variable that exhibits the largest time-varying movement is yet again the Mkt.RF variable, with a beta value of 1.101. An interesting evolution appears in the case of HML and CMA variables, which both exhibit good beta values, albeit negative. In the case of HML, the value of beta is -0.133, while in the case of CMA, the value of beta takes a value of -0.117. It is interesting to note that, this particular industry departs from the classical time-varying evolution of the Mkt.RF and RMW couple. It can be argued that the

Table 17. Hierarchical Bayesian Lasso-Triple Gamma Prior for the "Technology" Industry

Parameter	Mean	SD	Median	HPD 2.5%	HPD 97.5%	ESS
β_2 Intercept	-0.040	0.060	-0.006	-0.179	0.039	775
β_2 Mkt.RF	1.078	0.086	1.076	0.909	1.256	151
β_2 SMB	0.029	0.066	0	-0.069	0.194	256
β_2 HML	0.009	0.106	0	-0.225	0.272	218
β_2 RMW	0.393	0.117	0.377	0.185	0.677	313
β_2 CMA	0.013	0.102	0	-0.177	0.250	247
$\|\theta\|$ Intercept	0.004	0.008	0.001	0	0.019	505
$\|\theta\|$ Mkt.RF	0.013	0.005	0.012	0.003	0.023	301
$\|\theta\|$ SMB	0.014	0.008	0.014	0	0.028	228
$\|\theta\|$ HML	0.056	0.014	0.054	0.031	0.084	225
$\|\theta\|$ RMW	0.007	0.008	0.004	0	0.023	251
$\|\theta\|$ CMA	0.031	0.018	0.030	0	0.064	136
τ^2 Intercept	0.036	0.148	0	0	0.175	1075
τ^2 Mkt.RF	0.314	0.525	0.103	0	1.323	1891
τ^2 SMB	0.047	0.211	0	0	0.222	1398
τ^2 HML	0.042	0.181	0	0	0.220	2096
τ^2 RMW	0.197	0.406	0.043	0	0.923	1322
ξ^2 Intercept	0.037	0.187	0	0	0.169	2842
ξ^2 Mkt.RF	0.087	0.263	0.006	0	0.468	1862
ξ^2 SMB	0.093	0.259	0.006	0	0.503	1595
ξ^2 HML	0.208	0.421	0.044	0	1.003	1183
ξ^2 RMW	0.051	0.192	0.001	0	0.271	2017
ξ^2 CMA	0.149	0.366	0.018	0	0.742	1277
λ^2 Intercept	0.486	0.695	0.218	0	1.891	2757
λ^2 Mkt.RF	0.221	0.428	0.049	0	1.035	1112
λ^2 SMB	0.476	0.673	0.211	0	1.834	2770
λ^2 HML	0.460	0.681	0.188	0	1.774	2578
λ^2 RMW	0.327	0.543	0.106	0	1.385	1486
λ^2 CMA	0.456	0.676	0.192	0	1.809	2675
κ^2 Intercept	0.495	0.714	0.222	0	1.935	2636
κ^2 Mkt.RF	0.438	0.643	0.191	0	1.702	2677
κ^2 SMB	0.444	0.645	0.184	0	1.724	2573
κ^2 HML	0.309	0.518	0.102	0	1.336	2079
κ^2 RMW	0.473	0.677	0.199	0	1.781	2943
κ^2 CMA	0.389	0.597	0.141	0	1.605	2848
ξ	0.161	0.052	0.154	0.060	0.259	565
$c\xi$	0.378	0.071	0.389	0.240	0.496	760
τ	0.133	0.057	0.129	0.027	0.235	80
$c\tau$	0.383	0.068	0.393	0.246	0.490	872
$\kappa^2 B$	6712	59875	361.050	0.016	12712.589	404
$\lambda^2 B$	233.900	2179	7.611	0	374.853	340
σ^2	1.854	0.123	1.850	1.619	2.095	706
C_0	1.929	0.718	1.833	0.659	3.352	5000

Notes: The specifications are similar to those in Table 2.

TABLE 18. Through Stochastic Volatility, "Technology" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	-0.017	0.051	0	-0.151	0.049	609
β_2 Mkt.RF	1.096	0.074	1.092	0.961	1.252	206
β_2 SMB	0.009	0.050	0	-0.072	0.143	172
β_2 HML	0.003	0.076	0	-0.194	0.178	313
β_2 RMW	0.351	0.089	0.351	0.211	0.566	147
β_2 CMA	0.002	0.090	0	-0.203	0.221	272
$\|\theta\|$ Intercept	0.007	0.011	0.001	0	0.033	165
$\|\theta\|$ Mkt.RF	0.010	0.004	0.010	0.003	0.019	287
$\|\theta\|$ SMB	0.015	0.007	0.014	0	0.028	208
$\|\theta\|$ HML	0.043	0.014	0.041	0.017	0.069	85
$\|\theta\|$ RMW	0.003	0.006	0.001	0	0.016	126
$\|\theta\|$ CMA	0.032	0.014	0.031	0.008	0.062	130
τ^2 Intercept	0.413	16.790	0	0	0.207	5000
τ^2 Mkt.RF	40.307	1744	1.258	0.062	25.781	5000
τ^2 SMB	0.945	55.680	0	0	0.163	5000
τ^2 HML	0.253	3.404	0	0	0.406	5000
τ^2 RMW	10.127	321.100	0.218	0	7.364	2671
τ^2 CMA	0.934	49.840	0	0	0.336	5000
ξ^2 Intercept	0.004	0.037	0	0	0.009	2657
ξ^2 Mkt.RF	0.568	39.620	0	0	0.018	5000
ξ^2 SMB	0.022	0.741	0.001	0	0.027	5000
ξ^2 HML	0.033	0.318	0.003	0	0.084	4554
ξ^2 RMW	0.011	0.490	0	0	0.003	5000
ξ^2 CMA	0.035	0.442	0.002	0	0.060	4316
ξ	0.123	0.046	0.116	0.043	0.212	319
τ	0.090	0.038	0.083	0.025	0.163	164
$\kappa^2 B$	332.230	484.900	152.084	0	1262.218	3428
$\lambda^2 B$	7.905	17.790	1.756	0	36.018	3411
μ	0.464	0.115	0.464	0.243	0.695	1114
ϕ	0.853	0.080	0.869	0.701	0.970	50
σ^2	0.101	0.055	0.091	0.018	0.210	63

Notes: The specifications are similar to those in Table 2.

TABLE 19. Results for the Generalized Linear Model for the "Health" Industry (HLT)

Parameter	**Estimate (SE)**
Intercept	0.332^{***} (0.079)
Mkt.RF	0.984^{***} (0.023)
SMB	-0.059^{**} (0.034)
HML	-0.208^{***} (0.040)
RMW	-0.405^{***} (0.048)
CMA	-0.292^{***} (0.068)

Notes: Standard errors are reported in parentheses. *, **, *** indicate significance at the 90%, 95%, and 99% levels, respectively. The specifications are similar to those in Table 1.

specifics of the industry play a significant role. Given the construction of the factors, it can be noted that the baseline for this industry is large or very large companies. Hence, this resulted

Table 20. Hierarchical Bayesian Lasso for the “Health” Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.334	0.182	0.332	-0.026	0.702	1572
β_2 Mkt.RF	1.101	0.218	1.092	0.666	1.531	158
β_2 SMB	0.023	0.152	0.008	-0.246	0.343	326
β_2 HML	-0.133	0.249	-0.107	-0.676	0.322	256
β_2 RMW	0.061	0.169	0.037	-0.258	0.411	631
β_2 CMA	-0.117	0.277	-0.087	-0.702	0.417	414
$\|\theta\|$ Intercept	0.013	0.012	0.010	0	0.036	1925
$\|\theta\|$ Mkt.RF	0.037	0.012	0.036	0.014	0.061	265
$\|\theta\|$ SMB	0.023	0.010	0.022	0.006	0.045	408
$\|\theta\|$ HML	0.057	0.017	0.056	0.026	0.092	320
$\|\theta\|$ RMW	0.016	0.012	0.013	0	0.038	690
$\|\theta\|$ CMA	0.074	0.039	0.072	0.008	0.143	100
τ^2 Intercept	0.372	0.807	0.176	0	1.229	1471
τ^2 Mkt.RF	0.681	0.829	0.442	0.049	1.987	862
τ^2 SMB	0.266	0.605	0.092	0	0.972	1498
τ^2 HML	0.313	0.678	0.127	0	1.224	1083
τ^2 RMW	0.261	0.558	0.103	0	0.982	1511
τ^2 CMA	0.346	0.832	0.133	0	1.285	1354
ξ^2 Intercept	0.005	0.010	0.002	0	0.017	1230
ξ^2 Mkt.RF	0.006	0.011	0.003	0	0.021	1292
ξ^2 SMB	0.005	0.010	0.002	0	0.018	1446
ξ^2 HML	0.007	0.012	0.004	0	0.022	1634
ξ^2 RMW	0.005	0.014	0.002	0	0.018	1696
ξ^2 CMA	0.008	0.012	0.005	0	0.026	702
$\kappa^2 B$	572.260	488.500	440.002	11.377	1510.943	615
$\lambda^2 B$	10.957	11.100	7.630	0.276	31.378	615
σ^2	5.631	0.352	5.616	4.972	6.336	1151
C_0	2.120	0.772	2.038	0.723	3.661	5000

Notes: The specifications are similar to those in Table 2.

in negative HML, since such companies are already expensive to own thus, leaving little room for retail investors to include in their portfolios.

Another interesting aspect comes in the form of the Investment factor or CMA variable, which shows a strong yet negative time-varying effect. An argument, in this case, can be advanced in the form of aggressive investment on the part of the companies especially in the R&D department to develop new drugs or improve the ones already available. Given the factor construction and the industry specifics, such results may not be a surprise to most readers. Nevertheless, it is an interesting perspective to see a divergence from the already classical couple of RMW and Mkt.RF variables. Similarly, the remainder of the variables exhibits time-varying effects throughout the period of interest, albeit at a lower amplitude. The graphical representations of the results can be viewed in Figure 31 and Figure 32.

A similar evolution as in the case of the previous industries can be observed. For instance, in the case of the Mkt.RF variable, the troughs in the 1980s and 2008-2010 period is in line with the evolution of previous industries. Another aspect that is worth noting, is the fact that, as in the case of the “Technology” Industry, after the 2010 mark, the companies within both sectors have dramatically increased their size. This can be argued not only by the fact that the specifics of both industries favor large-size corporations over smaller-sized companies but also by the fact that the “Health” Industry has a multitude of entry barriers and regulations.

Nevertheless, we will either confirm or adjust our opinion based on the following results from the remaining models. The next model of interest comes in the form of a Hierarchical Bayesian Lasso-ridge prior, which makes use of a ridge prior to better approximate the time-varying effects of the aforementioned variables. The results obtained can be viewed in Table 21.

Table 21. Hierarchical Bayesian Lasso-ridge prior "Health" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.321	0.166	0.324	-0.028	0.652	2164
β_2 Mkt.RF	0.898	0.164	0.901	0.566	1.227	170
β_2 SMB	0.050	0.168	0.039	-0.267	0.382	316
β_2 HML	-0.055	0.235	-0.060	-0.495	0.426	295
β_2 RMW	0.072	0.179	0.057	-0.237	0.456	618
β_2 CMA	-0.126	0.259	-0.131	-0.652	0.365	719
$\lvert\theta\rvert$ Intercept	0.015	0.014	0.012	0	0.044	1676
$\lvert\theta\rvert$ Mkt.RF	0.034	0.012	0.033	0.010	0.059	269
$\lvert\theta\rvert$ SMB	0.027	0.012	0.025	0.008	0.052	425
$\lvert\theta\rvert$ HML	0.059	0.019	0.058	0.025	0.098	296
$\lvert\theta\rvert$ RMW	0.019	0.014	0.017	0	0.045	619
$\lvert\theta\rvert$ CMA	0.109	0.037	0.109	0.035	0.179	162
σ^2	5.537	0.356	5.530	4.870	6.258	1471
C_0	2.142	0.787	2.035	0.736	3.665	5000

Notes: The specifications are similar to those in Table 2.

The results obtained are in line with the case of the Hierarchical Bayesian Lasso, albeit with a small difference. The largest time-varying effect can be observed in the case of Mkt.RF, which has a beta value of 0.898, followed by the CMA variable with a negative beta value of -0.126. It is interesting to note that, compared to the previous model, the beta value of Mkt.RF has decreased, while on the other hand, the CMA variable remains significant. A significant departure from the previous model comes in the form of the HML variable, which suffered a large decrease in the value of its beta. Nevertheless, it can still be claimed that all the variables employed show a degree of time-varying effects. It can be argued that the same conditions and explanations apply to this model. The graphical representation of the results can be viewed in Figure 33 and Figure 34.

Nevertheless, it can be noted the negative influence of the investment variable, CMA and an explanation can be advanced in this regard. Given the nature of this industry and the latest and still ongoing pandemic crisis, the "Health" Industry started aggressive investment operations during the 2020-2022 period, to provide new drugs and vaccines to stop the pandemic. To this end, we argue that the R&D expenses allotted to the development of a possible cure for the Coronavirus, lead to a negative influence of this factor. Albeit, a temporary influence as the drugs and vaccines that have been approved contributed positively to the profitability and share price increases of a select number of companies. The next model employed comes in the form of the Hierarchical Bayesian Lasso-Double Gamma prior, which makes use of 2 priors to better capture and approximate the time-varying effects of the variables employed. The results obtained after applying this model are in Table 22.

The results obtained indicate that the variable which exhibits the largest time-varying effect is Mkt.RF, with a beta value of 1.157. It is also interesting to note that the previously significant variables, HML and CMA suffered a significant decline. Nevertheless, these variables remain significant, with the beta value of CMA at a negative -0.045, followed by the HML variable with a beta value of -0.037. A similar evolution as in the case of the previous models can be observed in the case of the Double Gamma prior. Namely, the negative impact of high R&D costs and perhaps also investments into new lines of production for either drugs or vaccines.

TABLE 22. Hierarchical Bayesian Lasso-Double Gamma Prior for the "Health" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.273	0.167	0.284	-0.021	0.554	482
β_2 Mkt.RF	1.157	0.214	1.141	0.769	1.576	162
β_2 SMB	0.007	0.079	0	-0.178	0.188	349
β_2 HML	-0.037	0.144	0	-0.430	0.192	206
β_2 RMW	-0.004	0.074	0	-0.162	0.125	657
β_2 CMA	-0.045	0.196	0	-0.504	0.335	289
$\lvert\theta\rvert$ Intercept	0.006	0.010	0.001	0	0.026	392
$\lvert\theta\rvert$ Mkt.RF	0.035	0.013	0.034	0.008	0.060	261
$\lvert\theta\rvert$ SMB	0.023	0.011	0.021	0.006	0.046	226
$\lvert\theta\rvert$ HML	0.055	0.017	0.053	0.025	0.089	172
$\lvert\theta\rvert$ RMW	0.006	0.009	0.003	0	0.024	252
$\lvert\theta\rvert$ CMA	0.087	0.042	0.088	0.004	0.156	100
τ^2 Intercept	7.057	161.410	0.129	0	5.317	3889
τ^2 Mkt.RF	34.715	1102.900	1.301	0.042	26.328	5000
τ^2 SMB	0.423	7.212	0	0	0.361	4677
τ^2 HML	0.453	6.095	0	0	0.790	2324
τ^2 RMW	0.189	2.672	0	0	0.314	5000
τ^2 CMA	1.201	25.183	0	0	1.134	4270
ξ^2 Intercept	0.004	0.034	0	0	0.009	4750
ξ^2 Mkt.RF	0.039	0.610	0.003	0	0.090	5000
ξ^2 SMB	0.034	0.457	0.001	0	0.065	5000
ξ^2 HML	0.134	3.025	0.006	0	0.157	2808
ξ^2 RMW	0.005	0.075	0	0	0.009	5000
ξ^2 CMA	0.105	1.118	0.011	0	0.249	2975
ξ	0.122	0.043	0.116	0.048	0.203	687
τ	0.096	0.041	0.088	0.033	0.182	161
κ_B^2	202.779	332.510	77.940	0	843.486	2980
λ_B^2	7.987	18.760	1.924	0	34.689	2109
σ^2	5.644	0.368	5.622	4.951	6.376	709
C_0	2.130	0.778	2.034	0.741	3.688	5000

Notes: The specifications are similar to those in Table 2.

Nevertheless, it is clear that due to the large size of the corporations, especially after the 2010 mark, the "value" investment opportunities are lacking. Thus, as the HML indicator suggests, given the entry barrier and the large size of the companies, certain variables are perhaps better at capturing the price formation mechanism in the case of this particular industry. We argue that, due to such considerations, the observation advanced in Fama and French (2015), namely that the HML or "value" factor is redundant can be confirmed.

To this end, we can also suggest that not only the evolution of companies after the 2010 mark is the lone culprit but also the general macroeconomic evolutions and the economic policies adopted. The graphical representation of those results can be viewed in Figure 35 and Figure 36. The next model, Hierarchical Bayesian Lasso-triple gamma prior, makes use of 3 priors to better capture and approximate the time-varying effects of the selected variables. The results obtained after running the model can be viewed in Table 23.

As in the case of the previous models, the Mkt.RF variable exhibits the highest time-varying effect, with a beta value of 1.194. Following is the CMA variable with a negative beta value of -0.045. In some respects, one may argue that those results are similar to the ones of the

TABLE 23. Hierarchical Bayesian Lasso-Triple Gamma Prior for the "Health" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.252	0.172	0.267	-0.024	0.549	240
β_2 Mkt.RF	1.194	0.223	1.173	0.757	1.626	158
β_2 SMB	0.008	0.097	0	-0.214	0.244	215
β_2 HML	-0.080	0.191	-0.003	-0.621	0.178	162
β_2 RMW	0.004	0.082	0	-0.151	0.176	594
β_2 CMA	-0.045	0.181	0	-0.507	0.270	346
$\lvert\theta\rvert$ Intercept	0.008	0.011	0.003	0	0.029	288
$\lvert\theta\rvert$ Mkt.RF	0.038	0.012	0.037	0.015	0.062	338
$\lvert\theta\rvert$ SMB	0.023	0.010	0.021	0.007	0.043	241
$\lvert\theta\rvert$ HML	0.059	0.018	0.057	0.025	0.096	214
$\lvert\theta\rvert$ RMW	0.007	0.009	0.004	0	0.025	318
$\lvert\theta\rvert$ CMA	0.079	0.044	0.079	0.005	0.159	57
τ^2 Intercept	0.153	0.361	0.019	0	0.758	1952
τ^2 Mkt.RF	0.350	0.549	0.132	0	1.499	1472
τ^2 SMB	0.048	0.185	0	0	0.276	1028
τ^2 HML	0.076	0.241	0.001	0	0.428	1185
τ^2 RMW	0.040	0.184	0	0	0.209	2530
τ^2 CMA	0.062	0.220	0	0	0.356	1624
ξ^2 Intercept	0.033	0.154	0	0	0.168	1911
ξ^2 Mkt.RF	0.134	0.334	0.016	0	0.698	1746
ξ^2 SMB	0.087	0.249	0.007	0	0.451	1789
ξ^2 HML	0.163	0.363	0.028	0	0.795	1500
ξ^2 RMW	0.038	0.181	0	0	0.162	1733
ξ^2 CMA	0.184	0.368	0.036	0	0.885	903
λ^2 Intercept	0.369	0.578	0.137	0	1.505	2288
λ^2 Mkt.RF	0.175	0.367	0.034	0	0.832	1453
λ^2 SMB	0.449	0.641	0.192	0	1.792	2709
λ^2 HML	0.446	0.681	0.188	0	1.743	2736
λ^2 RMW	0.496	0.715	0.226	0	1.937	2978
λ^2 CMA	0.447	0.661	0.193	0	1.702	2748
κ^2 Intercept	0.488	0.695	0.212	0	1.878	2979
κ^2 Mkt.RF	0.397	0.607	0.166	0	1.557	2469
κ^2 SMB	0.452	0.656	0.195	0	1.794	2848
κ^2 HML	0.367	0.602	0.131	0	1.496	2232
κ^2 RMW	0.517	0.720	0.239	0	2.024	2976
κ^2 CMA	0.350	0.552	0.131	0	1.480	2398
ξ	0.163	0.052	0.158	0.067	0.266	619
$c\xi$	0.381	0.067	0.389	0.257	0.495	984
τ	0.143	0.053	0.138	0.053	0.251	211
$c\tau$	0.383	0.067	0.392	0.250	0.491	740
κ_B^2	1137.500	626180.960	5131.940	60.023	3750.787	495
λ_B^2	259.350	2114	710.206	0.001	444.908	414
σ^2	5.625	0.361	5.615	4.947	6.344	1074
C_0	2.127	0.777	2.028	0.821	3.695	5000

Notes: The specifications are similar to those in Table 2.

previous Double Gamma model, with the main difference being the HML variable, which yields a negative beta value of just -0.08. Given this, one may note that the predominant couple in the case of the "Health" Industry is not the already too common Mkt.RF and RMW variables, but the Mkt.RF and CMA variables. Given that the results obtained for this model are in line with the results of the previous models, we can argue that the economic explanations are similar.

The CMA and SMB variables exhibit the same evolution, especially after the 2010 mark, confirming the fact that this specific industry revolves around large-size corporations. It is also important to note that the Mkt.RF variable exhibits an upward trend starting from 2015 up to the 2020 mark. Given the exogenous shock of the 2020 pandemic, a decrease in the excess return of the market can be observed.

Although, one may note that the impact of the pandemic was minimal and that in 2022, the upward trend exhibited before resumed its steady advance. In the case of the Investment factor, CMA, after the 2020 peak, it can be noted that a downward trend has been resumed. This may be caused by the fact that during the 2020-2021 period, the new drugs and vaccines necessary to combat the pandemic have been developed. The graphical representation of the results can be viewed in Figure 37 and Figure 38. Lastly, the Through Stochastic Volatility model is employed in the "Health" Industry. The results obtained after running the model can be viewed in Table 24.

As was the case in the previous industries concerning the Through Stochastic Volatility model, the most relevant variable remains the Mkt.RF variable. In this case, with a beta value of 1.199, followed by the HML variable with a negative beta value of -0.064. It is interesting to note that for the first time, the Through Stochastic Volatility model does not capture the time-varying effects of the RMW variable at the same intensity and amplitude as was the case in the previous industries. This time, the RMW variable exhibits a small degree of time-varying effects, with a beta value of just 0.015.

With this in mind, we can yet again confirm that this model, Through Stochastic Volatility, attaches larger importance to the excess return of the market factor, Mkt.RF. Nevertheless, it is important to note that this particular industry has a preference for large-size corporations and at the same time, a larger appetite for R&D expenses that can be reflected by the CMA variable. Although an important industry in any economy, the many entry barriers, represented both by the high price of owning such a company and by the fact that, a certain number of permits and regulations have to be fulfilled, heavily impact the HML, or the "value" factor.

As the argument advanced by Fama and French (2015) provided us with some insight as to why this factor became redundant, especially in this post-GFC environment we argue that there are several factors besides the ones already mentioned. We argue that not only the general environment may pose a significant influence, but also the particularities of every industry. This was especially notable in the case of the "Technology" and "Health" Industries, both of which had a preference for large-size corporations. The graphical representation of the results obtained after running the model can be viewed in Figure 39 and Figure 40.

5.5. **Results for the "Other" Industries.** The last Industry tested is named the "Other" Industry or Industries. This name was given to differentiate between the previously analyzed industries and, the ones that were intentionally left out. Thus, this last industry compiles the remaining ones, and, the same research methodology is used to see whether the factors of the 5FM are indeed time-varying. Similarly, to the previous industries, the first model employed is the GLM, after which the results of the shrinkTVP framework are discussed.

Similarly, to the previous industries, the results for the "Other" industry are in line with the results provided by the shrinkTVP framework. Namely, the most statistically relevant factors remain the Mkt.RF followed by the CMA and HML factors. Concerning the latter, we argue that a similar result can be observed in the case of the "Health" Industry, where the HML factor,

TABLE 24. Through Stochastic Volatility, "Health" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.302	0.183	0.321	-0.017	0.614	279
β_2 Mkt.RF	1.199	0.222	1.185	0.777	1.628	134
β_2 SMB	0.001	0.055	0	-0.127	0.138	501
β_2 HML	-0.064	0.177	0	-0.556	0.172	95
β_2 RMW	0.015	0.095	0	-0.128	0.208	411
β_2 CMA	-0.021	0.109	0	-0.273	0.212	441
$\|\theta\|$ Intercept	0.011	0.016	0.003	0	0.042	186
$\|\theta\|$ Mkt.RF	0.035	0.011	0.035	0.015	0.057	349
$\|\theta\|$ SMB	0.020	0.008	0.019	0.007	0.036	376
$\|\theta\|$ HML	0.060	0.017	0.059	0.029	0.093	216
$\|\theta\|$ RMW	0.008	0.010	0.004	0	0.028	236
$\|\theta\|$ CMA	0.027	0.023	0.019	0	0.075	101
τ^2 Intercept	7.178	190.400	0.160	0	6.153	5000
τ^2 Mkt.RF	29.952	502.400	1.440	0.064	31.736	4641
τ^2 SMB	0.733	22.040	0	0	0.177	5000
τ^2 HML	1.495	49.110	0	0	1.273	5000
τ^2 RMW	0.785	15.840	0	0	0.397	3678
τ^2 CMA	0.898	15.450	0	0	0.627	5000
ξ^2 Intercept	0.008	0.072	0	0	0.017	3235
ξ^2 Mkt.RF	0.024	0.193	0.002	0	0.064	5000
ξ^2 SMB	0.020	0.310	0.001	0	0.041	5000
ξ^2 HML	0.126	4.868	0.006	0	0.135	5000
ξ^2 RMW	0.005	0.050	0	0	0.013	5000
ξ^2 CMA	0.020	0.201	0.001	0	0.055	4480
ξ	0.129	0.046	0.123	0.045	0.216	743
τ	0.088	0.038	0.081	0.028	0.164	168
κ_B^2	272.880	413.800	116.279	0	1095.542	2893
λ_B^2	7.726	21.160	1.431	0	33.428	3153
μ	1.596	0.146	1.604	1.307	1.874	1435
ϕ	0.933	0.037	0.941	0.857	0.992	99
σ^2	0.036	0.023	0.030	0.005	0.083	73

Notes: The specifications are similar to those in Table 2.

TABLE 25. Results for the Generalized Linear Model for the "Other" Industry (OTH)

Parameter	**Estimate (SE)**
Intercept	0.332 (0.064)
Mkt. RF	0.981*** (0.015)
SMB	0.122*** (0.022)
HML	-0.036 (0.029)
RMW	0.443*** (0.030)
CMA	0.221*** (0.068)

Notes: Standard errors are reported in parentheses. *, **, *** indicates significance at the 90%, 95%, and 99% level, respectively. The specifications are similar to those in Table 1.

while statistically significant, exerted a negative influence over the price formation mechanism within the industry.

As can be seen more clearly in the shrinkTVP framework, the Mkt.RF factor showcases the most time-variable effect, followed by the CMA and HML factors which showcase an almost similar time-variable effect over time, albeit negative. As with the previous cases, the first model run is the Hierarchical Bayesian Lasso. The results obtained after running the model can be viewed in Table 26.

TABLE 26. Hierarchical Bayesian Lasso for the "Other" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.334	0.182	0.332	-0.026	0.702	1572
β_2 Mkt.RF	1.101	0.218	1.092	0.666	1.531	158
β_2 SMB	0.023	0.152	0.008	-0.246	0.343	326
β_2 HML	-0.130	0.249	-0.107	-0.676	0.322	256
β_2 RMW	0.061	0.169	0.037	-0.258	0.411	631
β_2 CMA	-0.120	0.277	-0.087	-0.702	0.417	414
abs(θ Intercept)	0.013	0.012	0.010	0	0.036	1925
abs(θ Mkt.RF)	0.037	0.012	0.036	0.014	0.061	265
abs(θ SMB)	0.023	0.010	0.022	0.006	0.045	408
abs(θ HML)	0.057	0.017	0.056	0.026	0.092	320
abs(θ RMW)	0.016	0.012	0.013	0	0.038	690
abs(θ CMA)	0.074	0.039	0.072	0.008	0.143	100
τ^2 Intercept	0.372	0.807	0.176	0	1.229	1471
τ^2 Mkt.RF	0.681	0.829	0.442	0.049	1.987	862
τ^2 SMB	0.266	0.605	0.092	0	0.972	1498
τ^2 HML	0.313	0.678	0.127	0	1.224	1083
τ^2 RMW	0.261	0.558	0.103	0	0.982	1511
τ^2 CMA	0.346	0.832	0.133	0	1.285	1354
ξ^2 Intercept	0.005	0.010	0.002	0	0.017	1230
ξ^2 Mkt.RF	0.006	0.011	0.003	0	0.021	1292
ξ^2 SMB	0.005	0.010	0.002	0	0.018	1446
ξ^2 HML	0.007	0.012	0.004	0	0.022	1634
ξ^2 RMW	0.005	0.014	0.002	0	0.018	1696
ξ^2 CMA	0.008	0.012	0.005	0	0.026	702
$\kappa^2 B$	572.300	488.530	440.000	11.377	1510.943	615
$\lambda^2 B$	10.960	11.095	7.630	0.276	31.378	615
σ^2	5.631	0.352	5.616	4.972	6.336	1151
C_0	2.120	0.772	2.038	0.723	3.661	5000

Notes: The specifications are similar to those in Table 2.

As was the case with the other industries analyzed, with the sole exception of the "Health" Industry, this so-called "Other" Industry follows a similar pattern. Namely, the couple of variables Mkt.RF and RMW, make a return. The results obtained suggest that Mkt.RF has the highest time-varying effect with a beta value of 1.101, followed by the RMW variable with a beta value of 0.061. As was the case with the other industries, excluding the "Health" Industry, this couple has the largest and amplest time-varying effects. This, as previously discussed can be attributed to the fact that both the excess return of the market and the profitability of the sector or industry can and are affected by the general economic outlook and various types of shocks both endogenous and exogenous.

As a consequence, due to their dynamic qualities, it comes as no surprise as this couple, yet again, accounts for the most time-varying effects. The graphical representations of the results obtained can be viewed in Figure 41 and Figure 42. It can be noted that the results are also in line with the previous industries, although certain differences should be noted. One may note that the Mkt.RF variable indicates that this "Other" Industry suffered a larger decline during the 2000 Dotcom bubble collapse than the other industries analyzed. Similarly, during the 2008-2010 GFC, this industry, as mirrored by the Mkt.RF indicator performed better and did not suffer a significant decline.

Another relevant aspect is showcased by the HML or the "value" factor. It can be observed that starting from the 2000 mark, up until 2010, investors had an opportunity to diversify their portfolios by including a large amount of "value" oriented companies in their portfolios. When comparing the results obtained for this particular HML variable, with the results obtained in the case of the previous industries, one may note that the redundancy previously discussed disappears. Nevertheless, before reaching a concluding comment on the results obtained for this industry, further models employed may provide a clearer picture. The next model employed comes in the form of the Hierarchical Bayesian Lasso-ridge prior, which makes use of a ridge before better capturing and estimating the time-varying effects of different parameters or variables. The results obtained after running the model can be viewed in Table 27.

Table 27. Hierarchical Bayesian Lasso-ridge prior for the "Other" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.321	0.166	0.324	-0.028	0.652	2164
β_2 Mkt.RF	0.898	0.164	0.901	0.566	1.227	170
β_2 SMB	0.050	0.168	0.039	-0.267	0.382	316
β_2 HML	-0.055	0.235	-0.060	-0.495	0.426	295
β_2 RMW	0.072	0.179	0.057	-0.237	0.456	618
β_2 CMA	-0.126	0.259	-0.131	-0.652	0.365	719
abs(θ Intercept)	0.015	0.014	0.012	0	0.044	1676
abs(θ Mkt.RF)	0.034	0.012	0.033	0.010	0.059	269
abs(θ SMB)	0.027	0.012	0.025	0.008	0.052	425
abs(θ HML)	0.059	0.019	0.058	0.025	0.098	296
abs(θ RMW)	0.019	0.014	0.017	0	0.045	619
abs(θ CMA)	0.109	0.037	0.109	0.035	0.179	162
σ^2	5.537	0.356	5.530	4.870	6.258	1471
C_0	2.142	0.787	2.035	0.736	3.665	5000

Notes: The specifications are similar to those in Table 2.

The results are not surprising, since this Industry seems to align with the previous ones. Thus, the largest and amplest time-varying effect can be noted in the case of the Mkt.RF variable, which has a beta value of 0.898. Following suit is the CMA variable with a beta value of 0.126. Nevertheless, it should be noted that the remainder of the variables exhibit a lower and less ample time-varying effect, with the RMW and HML variables having the third and fourth largest effects, with a beta value of 0.072 and 0.055. The graphical results are presented in Figure 43 and Figure 44, located in the Appendices section of the paper. As with the previous model, one may note that the evolutions of the previously discussed variables are similar. A few minor differences do appear and they pertain to an interesting discussion.

The main variable of interest in the case of this industry is still represented by the Mkt.RF variable but the "value" factor, HML shows an interesting evolution. As one may note, a sharp decrease occurred exactly at the 2000 mark, when the Dotcom bubble burst, following a very sharp upturn in the same year. An argument we advance is that the popularity of the tech sector did attract a large number of retail investors, especially at the peak of the bubble, in the 1999–2000-time frame. Nevertheless, after the bursting of the bubble, investors chose to diversify and followed a "value-oriented" investment strategy. It can also be noted that the peak was reached in the 2008 GFC moment. This further reinforces the notion that during an economic downturn, investors prefer to allocate their capital towards more "value" oriented companies or, divest away from companies that they perceive as risky. The next model carried out comes in the form of the Hierarchical Bayesian Lasso-Double Gamma prior, which makes use of 2 priors to better capture and estimate the effects of time-varying parameters and variables. The results obtained after running the model can be viewed in Table 28.

Table 28. Hierarchical Bayesian Lasso-Double Gamma prior for the "Other" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.273	0.167	0.284	-0.021	0.554	482
β_2 Mkt.RF	1.157	0.214	1.141	0.769	1.576	162
β_2 SMB	0.007	0.079	0	-0.178	0.188	349
β_2 HML	-0.037	0.144	0	-0.430	0.192	206
β_2 RMW	-0.004	0.074	0	-0.162	0.125	657
β_2 CMA	-0.045	0.196	0	-0.504	0.335	289
abs(θ Intercept)	0.006	0.010	0.001	0	0.026	392
abs(θ Mkt.RF)	0.035	0.013	0.034	0.008	0.060	261
abs(θ SMB)	0.023	0.011	0.021	0.006	0.046	226
abs(θ HML)	0.055	0.017	0.053	0.025	0.089	172
abs(θ RMW)	0.006	0.009	0.003	0	0.024	252
abs(θ CMA)	0.087	0.042	0.088	0.004	0.156	100
τ_2 Intercept	7.057	161.408	0.129	0	5.317	3889
τ_2 Mkt.RF	34.715	1102.920	1.301	0.042	26.328	5000
τ_2 SMB	0.423	7.212	0	0	0.361	4677
τ_2 HML	0.453	6.095	0	0	0.790	2324
τ_2 RMW	0.189	2.672	0	0	0.314	5000
τ_2 CMA	1.201	25.183	0	0	1.134	4270
ξ_2 Intercept	0.004	0.034	0	0	0.009	4750
ξ_2 Mkt.RF	0.039	0.610	0.003	0	0.090	5000
ξ_2 SMB	0.034	0.457	0.001	0	0.065	5000
ξ_2 HML	0.134	3.025	0.006	0	0.157	2808
ξ_2 RMW	0.005	0.075	0	0	0.009	5000
ξ_2 CMA	0.105	1.118	0.011	0	0.249	2975
ξ	0.122	0.043	0.116	0.048	0.203	687
τ	0.096	0.041	0.088	0.033	0.182	161
κ_2 B	202.779	332.509	77.940	0	843.486	2980
λ_2 B	7.987	18.760	1.924	0	34.689	2109
σ^2	5.644	0.368	5.622	4.951	6.376	709
C_0	2.130	0.778	2.034	0.741	3.688	5000

Notes: The specifications are similar to those in Table 2.

Similarly, to the Hierarchical Bayesian Lasso-ridge prior, the main time-varying effect can be observed in the case of the Mkt.RF variable, which has a beta value of 1.157. Followed by the CMA variable, which has a negative beta value of -0.045. It can also be noted that the remainder of the variables does exhibit a degree of time-varying effect albeit, at a lower amplitude than the aforementioned variables. The results obtained can be viewed in the graphical form in Figure 45 and Figure 46. The results obtained in the Double Gamma prior model, suggest that the previous explanations continue to hold, albeit one may note the fact that the influences exercised by the CMA, HML, and RMW are negative.

This may be caused by the fact that, given the amalgamation of companies that can be found in this industry, a unitary industry specific is difficult to identify. Given this, certain companies that are more reliant on investment expenses or that register losses may negatively influence the efficiency of the factors used. The following model comes in the form of Hierarchical Bayesian Lasso-Triple Gamma prior, which makes use of 3 priors to better capture and showcase the time-varying effects of variables and parameters. The results obtained can be viewed in Table 29.

The results obtained suggest that as in the case of the Double Gamma prior, yet again the time-varying effects are most notable in the case of the Mkt.RF variable. The beta value of Mkt.RF is 1.194, followed by the CMA variable with a negative beta value of -0.045. The other variables do indeed present time-varying effects albeit, at a lower amplitude than the aforementioned variables. As it can be noted, the Tripe Gamma prior tends to capture the time variability of the HML variable better than the previous models. It can be observed that the peak is more pronounced, especially in the year 2008. This result may question the redundancy of this factor altogether as it can be observed that it successfully captures the rotation that most investors are forced to do in times of economic downturn.

This rotation so to speak is the choice to diversify away from "growth" and cyclical stocks that perform best when the economy is expanding but perform poorly during contractions or economic recessions. The rotation from these types of stocks to the "value" or non-cyclical related stocks allows investors to protect their wealth and, is a potential asset price driver in the case of "value" stocks. Given this, the redundancy of this factor can be a further research subject on its own. The graphical representation of the results obtained can be viewed in Figure 48 and Figure 49. Lastly, the Through Stochastic Volatility model is employed in the case of Other Industries. The results obtained can be viewed in Table 30.

The results obtained showcase that, as was the case with this particular model in the case of the previously tested industries, the Mkt.RF variable remains the strongest. In this case, the Mkt.RF variable shows the strongest and amplest time-varying effects, with a beta value of 1.199. In an interesting change, the RMW variable exhibits a reduced time-varying effect, with a beta value of 0.015. An interesting aspect is that the second strongest time-varying effect appears in the HML variable, with a negative beta value of -0.064. The remainder of the variables exhibit a certain degree of time variable effects, albeit at a lower and smaller amplitude. Given those results, we argue that in the case of the "Other" Industries, the leading factors are the Mkt.RF and the HML factors.

The results also suggest that the HML or "value" factor, has the potential to identify and pinpoint the risk-averse allocation decision of investors during economic downturns and, perhaps, the preference of certain investors who adopt a "value "oriented approach to incorporate in their portfolios companies at a significant discount. The results obtained can be viewed graphically in Figure 46 and Figure 47.

6. Comments regarding the results

This section of the paper is reserved for a discussion regarding the results obtained after performing the methodology provided within the "shrinkTVP" package. This discussion is further divided into two parts namely: the first part considers the results obtained from a macroeconomic perspective under which the specifics of each industry are taken into account. The second topic regards the implication of such results for the paper at hand and also for the 5FM model.

6.1. Implications regarding the industries.

Starting with the results obtained, we argue that the couple formed by the "Consumer" and "Manufacturing" Industries show similar tendencies and evolutions over time especially due to similar industry specifics. The main argument in favor of this is the fact that both industries exhibit a cyclical evolution in line with the macroeconomic background. As can be observed from the evolution of the Mkt.RF variable, both industries suffered setbacks during the recessionary periods of the 1980s, and early 2000s, and the GFC of 2008-2010. Furthermore, as can be observed by the profitability variable,

Table 29. Hierarchical Bayesian Lasso-Triple Gamma prior for the "Other" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.252	0.172	0.267	-0.024	0.549	240
β_2 Mkt.RF	1.194	0.223	1.173	0.757	1.626	158
β_2 SMB	0.008	0.097	0	-0.214	0.244	215
β_2 HML	-0.080	0.191	-0.003	-0.621	0.178	162
β_2 RMW	0.004	0.082	0	-0.151	0.176	594
β_2 CMA	-0.045	0.181	0	-0.507	0.270	346
abs(θ Intercept)	0.008	0.011	0.003	0	0.029	288
abs(θ Mkt.RF)	0.038	0.012	0.037	0.015	0.062	338
abs(θ SMB)	0.023	0.010	0.021	0.007	0.043	241
abs(θ HML)	0.059	0.018	0.057	0.025	0.096	214
abs(θ RMW)	0.007	0.009	0.004	0	0.025	318
abs(θ CMA)	0.079	0.044	0.079	0.005	0.159	57
τ_2 Intercept	0.153	0.361	0.019	0	0.758	1952
τ_2 Mkt.RF	0.350	0.549	0.132	0	1.499	1472
τ_2 SMB	0.048	0.185	0	0	0.276	1028
τ_2 HML	0.076	0.241	0.001	0	0.428	1185
τ_2 RMW	0.040	0.184	0	0	0.209	2530
τ_2 CMA	0.062	0.220	0	0	0.356	1624
ξ_2 Intercept	0.033	0.154	0	0	0.168	1911
ξ_2 Mkt.RF	0.134	0.334	0.016	0	0.698	1746
ξ_2 SMB	0.087	0.249	0.007	0	0.451	1789
ξ_2 HML	0.163	0.363	0.028	0	0.795	1500
ξ_2 RMW	0.038	0.181	0	0	0.162	1733
ξ_2 CMA	0.184	0.368	0.036	0	0.885	903
λ_2 Intercept	0.369	0.578	0.137	0	1.505	2288
λ_2 Mkt.RF	0.175	0.367	0.034	0	0.832	1453
λ_2 SMB	0.449	0.641	0.192	0	1.792	2709
λ_2 HML	0.446	0.681	0.188	0	1.743	2736
λ_2 RMW	0.496	0.715	0.226	0	1.937	2978
λ_2 CMA	0.447	0.661	0.193	0	1.702	2748
κ_2 Intercept	0.488	0.695	0.212	0	1.878	2979
κ_2 Mkt.RF	0.397	0.607	0.166	0	1.557	2469
κ_2 SMB	0.452	0.656	0.195	0	1.794	2848
κ_2 HML	0.367	0.602	0.131	0	1.496	2232
κ_2 RMW	0.517	0.720	0.239	0	2.024	2976
κ_2 CMA	0.350	0.552	0.131	0	1.480	2398
ξ	0.163	0.052	0.158	0.067	0.266	619
$c\xi$	0.381	0.067	0.389	0.257	0.495	984
τ	0.143	0.053	0.138	0.053	0.251	211
$c\tau$	0.383	0.067	0.392	0.250	0.491	740
κ_2 B	1137.560	6181	131.950	0.023	3750.787	495
λ_2 B	259.350	2114	10.206	0.001	444.908	414
σ^2	5.625	0.361	5.615	4.947	6.344	1074
C_0	2.127	0.777	2.028	0.821	3.695	5000

Notes: The specifications are similar to those in Table 2.

TABLE 30. Through Stochastic Volatility, "Other" Industry

Parameter	**Mean**	**SD**	**Median**	**HPD 2.5%**	**HPD 97.5%**	**ESS**
β_2 Intercept	0.302	0.183	0.321	-0.017	0.614	279
β_2 Mkt.RF	1.199	0.222	1.185	0.777	1.628	134
β_2 SMB	0.001	0.055	0	-0.127	0.138	501
β_2 HML	-0.064	0.177	0	-0.556	0.172	95
β_2 RMW	0.015	0.095	0	-0.128	0.208	411
β_2 CMA	-0.021	0.109	0	-0.273	0.212	441
abs(θ Intercept)	0.011	0.016	0.003	0	0.042	186
abs(θ Mkt.RF)	0.035	0.011	0.035	0.015	0.057	349
abs(θ SMB)	0.020	0.008	0.019	0.007	0.036	376
abs(θ HML)	0.060	0.017	0.059	0.029	0.093	216
abs(θ RMW)	0.008	0.010	0.004	0	0.028	236
abs(θ CMA)	0.027	0.023	0.019	0	0.075	101
τ_2 Intercept	7.178	190.431	0.160	0	6.153	5000
τ_2 Mkt.RF	29.952	502.411	1.440	0.064	31.736	4641
τ_2 SMB	0.733	22.040	0	0	0.177	5000
τ_2 HML	1.495	49.114	0	0	1.273	5000
τ_2 RMW	0.785	15.842	0	0	0.397	3678
τ_2 CMA	0.898	15.448	0	0	0.627	5000
ξ_2 Intercept	0.008	0.072	0	0	0.017	3235
ξ_2 Mkt.RF	0.024	0.193	0.002	0	0.064	5000
ξ_2 SMB	0.020	0.310	0.001	0	0.041	5000
ξ_2 HML	0.126	4.868	0.006	0	0.135	5000
ξ_2 RMW	0.005	0.050	0	0	0.013	5000
ξ_2 CMA	0.020	0.201	0.001	0	0.055	4480
ξ	0.129	0.046	0.123	0.045	0.216	743
τ	0.088	0.038	0.081	0.028	0.164	168
κ_2 B	272.882	413.787	116.279	0	1095.542	2893
λ_2 B	7.726	21.164	1.431	0	33.428	3153
μ	1.596	0.146	1.604	1.307	1.874	1435
ϕ	0.933	0.037	0.941	0.857	0.992	99
σ^2	0.036	0.023	0.030	0.005	0.083	73

Notes: The specifications are similar to those in Table 2.

RMW, both industries show declining profitability over the period. This can be mainly attributed to higher production costs and declining demand, especially during recessionary periods. Another relevant development that those particular industries face in the present time, regards the inflationary environment and supply shocks.

On the one hand, the industries at hand are facing increased prices in both energy and materials, while on the other hand, due to several exogenous shocks such as the pandemic crisis and the current geopolitical situation, the supply chains are affected. Another key element regarding the industries, or more precisely the "Manufacturing" Industry, is represented by the inclusion of both utilities and energy sectors within it. Arguably, given the current macroeconomic environment, it is interesting to witness the declining profitability of this Industry, as gauged by the RMW factor. We argue that given the proportion that "Manufacturing" occupies within the industry, the excess profit recorded by the energy sector does not manage to compensate for the increased costs of both energy and materials witnessed by the "Manufacturing" Industry. With this in mind, we argue that another element that may also hamper the beneficial effect of the Energy and Utilities sectors on the profitability of the industry, in general, comes in the form of government intervention and price ceilings. Given this, we argue that the results obtained for the "Consumer" and "Manufacturing" Industries are in line with the current macroeconomic development, and, to this end, we argue that the 5FM successfully captures the time-varying effects of factors on the price formation mechanism.

The results for the following 2 industries, "Technology", and "Health", can be discussed together from one perspective. This perspective refers to the current leading characteristic of the industry: large-size companies. To this end, taking the company size factor, SMB, into account, one may note that both industries are dominated by large-sized firms. This is even more evident when focusing on the post-GFC period, from 2010 onwards. Another similar aspect can be viewed through the excess return of the market variable, Mkt.RF has a similar evolution for both industries, especially during the occurrence of both endogenous and exogenous shocks. Nevertheless, certain differences appear, which can be linked to the different industry specifics.

For instance, taking into account the high R&D costs for the "Health" industry, our results suggest that during the pandemic crisis, firms in this domain of activity invested their resources in a rather aggressive manner. To this end, one may note that the investment factor, CMA, for this industry registered larger and positive time value effects over time. We argue that this is the result of the research and development strategy put together to develop new drugs and vaccines to stop the pandemic.

The same can be said for the "Technology" Industry, which also incorporates R&D services within it. Although the expenses recorded in this category are perhaps not as large as in the case of the "Health" Industry, we argue that they still represent an important characteristic of this industry.

Lastly, the results obtained for the "Other" Industry display a mixture of results that are mainly in line with the previously discussed industries. We argue that the results obtained in the case of this industry are determined by the large variation of industry specifics that are included within the industry. Nevertheless, as one may note, this particular plethora of industries exhibits a larger sensitivity to both shocks and macroeconomic evolutions. This can be further reinforced by the "mix" of sectors included in this Industry, such as Entertainment, Finance, Construction, or Mining.

Taking into account the Mkt.RF factor, we note that during the collapse of the Dotcom bubble, the "Other" industry suffered a large decline. On the other hand, during the GFC period, this industry suffered a marginal decline compared to the one in 2000. Nevertheless, it can be argued that during the 2020 pandemic crisis, the industry suffered large setbacks. Another different result can be seen in the case of the SMB factor, which presents a certain preference for smaller-sized companies for this particular industry. This trend can be traced back to the bursting of the Dotcom bubble in 2000.

We argue that given the large number of specifics that can be found within this industry, smaller-sized companies are more present. Given this, the largest time-varying effect over the period used was registered in the case of the excess market return, Mkt.RF factor. Followed by the profitability factor, RMW, and, lastly the size factor, SMB, and the investment factor, CMA. Those results suggest that given the time-varying effects of the aforementioned factors, their inclusion within the model is fully justified. Furthermore, we consider the excess market return and the profitability factors to be the main drivers concerning asset pricing. This argument can be enforced by the fact that investors demand a given level of return when exposed to additional risk and uncertainty.

Another element worth noting is that investors prefer to allocate their capital to companies that register a certain degree of profitability. Hence, it can be argued in favor of these factors as the main drivers of asset pricing within the 5FM. It is also important to note that, for all the analyzed industries, the "value" factor, HML, displayed a certain degree of time-variable effect. The most important milestone achieved by this factor is the fact that it can identify the moment when the market switches from a "bullish" stance to a "bearish" stance. Given this result, we consider that, while the factor itself shows a smaller impact on the asset pricing mechanism of the model, it can still provide valuable insight.

6.2. **Implications for the 5FM model.** Thus, we consider that given those aforementioned results, the 5FM proved its reliability. We argue that the 5FM model can capture the time-varying effects of the factors better than other more traditional models. Furthermore, given this ability, it can be argued that the model can be used to estimate the prices of stocks faithfully. Another important fact that is worth mentioning, is that the results obtained within this paper demonstrate a degree of similarity with the results obtained by Liammukda et al (2020). While the works of Liammukda et al (2020), aim to identify and observe the time-varying effects of the 5FM on the Japanese market, we aimed to create a similar analysis while using entire industries.

Furthermore, as the aforementioned authors note, the fact that the 5FM manages to capture both the positive and negative influence of the factors of the time is of crucial importance. As a consequence, it can be argued that when compared with the previously discussed models, namely the 3FM and the CAPM, the 5FM stands out due to this. With this in mind, we argue that a relevant implication for the efficiency of the 5FM is the fact that the initial or vanilla factor loading provides the best results while implemented in the case of developed markets.

Given the informational deficiencies and generally lower liquidity coupled with other factors, we consider that a different factor loading that is more in line with the market specifics could yield better results for emerging markets. Another point that we would like to discuss is the construction of the factors. We consider that given their construction that is heavily reliant on the quality of financial information, certain issues may arise in the case of emerging and frontier markets.

Considering that such markets present a tendency for lower-quality financial data, the efficiency of the model may be negatively impacted as was mentioned in the works of Foye and Valentinčič (2020). Another point that has a significant implication is whether the endogeneity effect may impact the relevance of the model and influence the pricing formation mechanism that the model provides. This specific topic may prove to be a good starting point for future research. With this in mind, we conclude that the 5FM remains a venerable tool that can be used to determine and faithfully price different assets using either the vanilla factor loadings in the case of developed markets or, an augmented factor loading in the case of emerging and frontier markets.

6.3. **Policy Implications.** Several policy implications can be derived from the results obtained. On the one hand, it can be argued that the implementation of the 5-Factor Model with time-varying parameters may provide a better understanding regarding the pricing of risk and of the potential sources of risk. Another key element worth mentioning regards the usage of such a model by institutional investors and fund managers and how such an approach may be beneficial for financial stability for both medium and long-term periods. It can also provide policymakers and regulators with a tool that may be used to approximate not only the perceived level of risk of an asset but also the different types of risks that may arise either from industry specifics or from the different evolution over time of the company itself.

Another key aspect worth mentioning is the fact that the results obtained after implementing such an approach can capture both general influences arising from the given risk factors, and also more specific influences to some extent. We consider that such influences are generated by certain specifics within the industry or sector of choice. With this in mind, we argue that this fact may provide certain advantages. For instance, considering the case of both regulators and investors, a tool that can provide and show how the given level of perceived risk evolves in a given industry or a general configuration could provide a better framework for decision-making. Another relevant aspect of interest especially for policymakers, may revolve around the specific levels of perceived risk surrounding key and important industries such as the banking industry. Having a tool that provides such relevant insight may ultimately aid in fulfilling the financial stability mandate of central banks and other key institutions. Nevertheless, several limitations of this study may require further and more in-depth research. Of these, we remember the current inability to exactly pinpoint whether the time-varying effects are generated exclusively by the general considerations or whether the specifics of each selected industry play a crucial role for such results.

Lastly, a relevant limitation may arise from the sensitivity of the model to the quality of financial information thus influencing the selected data employed in this study. While also making its implementation in different markets or industries an arduous process. Furthermore, we argue that a potential avenue of future research may be provided in the form of a non-linear approach to the model. Such an approach may provide users with a more detailed view regarding the sources of potential risk. While in the same time, it would explain more faithfully the evolution of perceived risk for each parameter and how the influence of the given parameters changes over time.

7. Conclusions

In conclusion, we can argue that the evolution of mainstream asset pricing was pioneered by the Capital Asset Pricing Model and the excess market return risk premium introduced by it. Given the pioneering work and path laid by this model, future research opportunities arose. Among those, our lenses focused on the 3 Factor and 5 Factor Models, with a preference for the latter. The main reason for focusing on this specific model is twofold. On the one hand, since it is constructed on a common ground with the theoretical background of the CAPM. As both models operate no clear distinction between the concept of risk and uncertainty and make use of the excess return of the market as the main factor.

The 5-factor Model attempted to explain and account for a portfolio's excess return derived from a multitude of risk factors. Given this theoretical background, the next step was to test the time-varying effects of the factors of the 5-Factor Model. To this end, we have employed the vanilla factors constructed in a 2x3 sort tested on the excess return of 5 large industries. The industries tested are the "Consumer" Industry, the "Manufacturing" Industry, the "Technology" Industry, the "Health" Industry, and lastly, the so-called "Other" Industry. Thus, the research objective of the paper was to test whether the factors of the vanilla 5-Factor Model exhibit time-varying effects over the analyzed period. The analyzed period starting in July 1963 and ending in June 2022, results in a number of 708 monthly observations. As mentioned before, the choice for monthly observations was enforced by the fact that the 5-Factor Model may prove a better tool for short- and medium-term portfolio management.

The methodology applied made use of the "shrinkTVP" package in R, which employs a TVPM-S model with 5 different priors to better estimate and capture the time-varying effects of the variables. Another great asset provided by the package comes in the form of graphical representations of the results which can present the time variability of the factors in a smoother graphical manner. With this in mind, the results obtained do indeed confirm the fact that the factors of the vanilla 5-Factor Model exhibit time variability over the observation period.

The factors that exhibit the largest and amplest time-varying effects are the Mkt.RF or the excess return of the market and the RMW or the Profitability factor. These results should come as no surprise given the fact that both factors can have a dynamic evolution over time. This can be especially true in the case of the excess market return which can present large fluctuations from market phase to market phase. Nevertheless, the profitability of companies may suffer variable degrees of fluctuations given the economic situation and the specifics of the industry. As a result, the dominant couple is represented by these variables. The only deviation from this norm was presented in the case of the "Health" and "Other" Industries where the Mkt.RF factor and the CMA or the Investment factor seem to be the dominant variables that exhibit the largest time-varying effects.

It may be hypothesized that the industry specifics are at fault for those results in the case of the aforementioned industries. This can be especially true in the case of the "Health" Industry, in which the resulting CMA factor indicates a rather aggressive investment pattern. To this end, we believe that in the case of this particular industry, the specifics of the industry have a large influence, especially due to the nature of investment expenses involved, such as the R&D effort in developing new drugs. Nevertheless, it can be stated with a great degree of confidence that the factors used in the 5-Factor Model do indeed present time variable effects over the observation period. Given this, perhaps further research opportunities may arise either in testing augmented flavors of the model or by expanding the number of industries to check whether the results detach from the now classical couple of Mkt.RF and RMW variables and, whether different industry specifics play a decisive role in the time-varying effects of those factors.

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Appendices

The following Appendices section is aimed at presenting the graphical results of the TVPM-S methodology employed. The resulting figures have been generated for each Prior employed and for each Industry. To maximize space, we have condensed the figures. Nevertheless, they are made available by the authors on request at the initial full size.

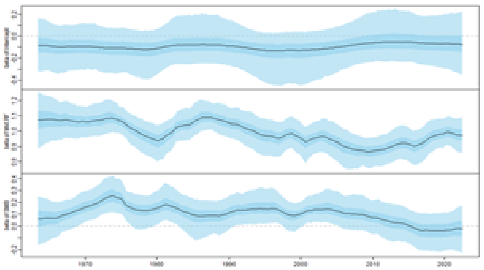

Figure 1, Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso, "Consumer" Sector

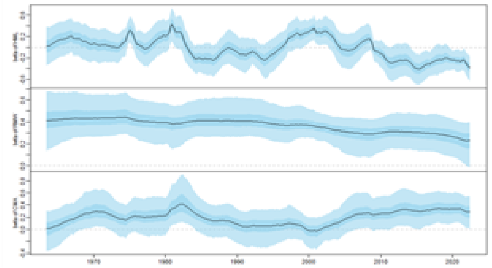

Figure 2, Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso, "Consumer" Sector

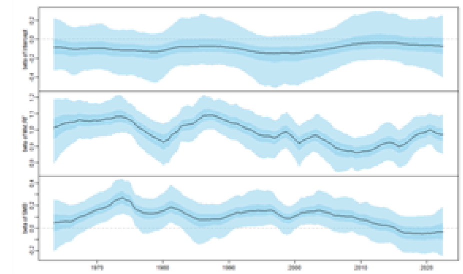

Figure 3, Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Ridge prior, CNS

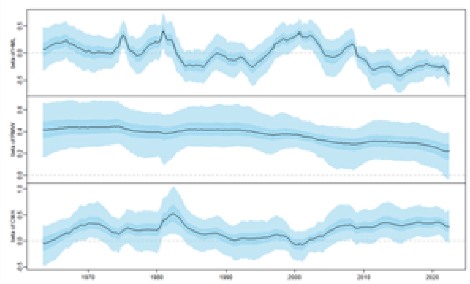

Figure 4, Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Ridge prior, CNS

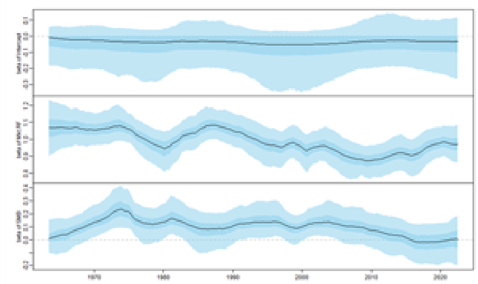

Figure 5, Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Double Gamma prior, CNS

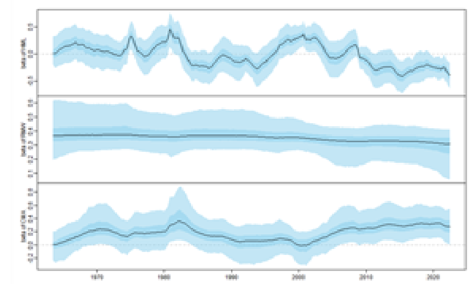

Figure 6, Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Double Gamma prior, CNS

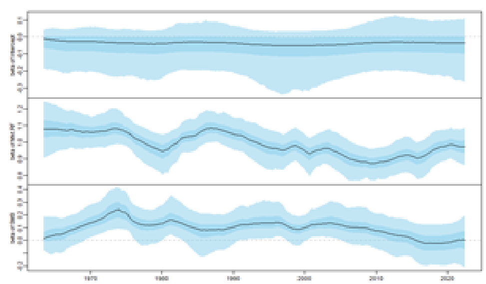

Figure 7, Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Triple Gamma prior, CNS

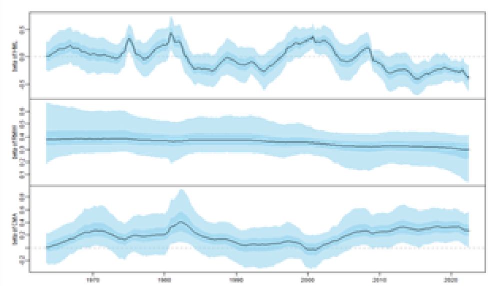

Figure 8, Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Triple Gamma prior, CNS

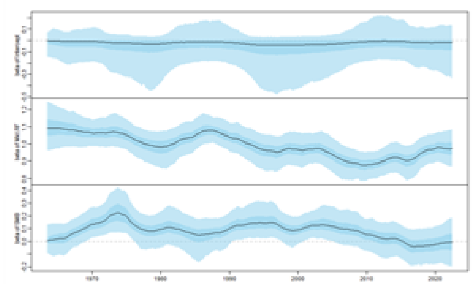

Figure 9, Time-varying effects of Intercept, SMB, and Excess Return factors Through Stochastic Volatility, CNS

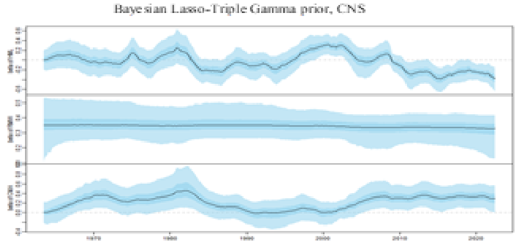

Figure 10, Time-varying effects of HML, RMW, and CMA factors Through Stochastic Volatility, CNS

Graphical representation of the results obtained, in a condensed form, of the TVPM-S methodology obtained for all the priors for the "Consumer" Industry.

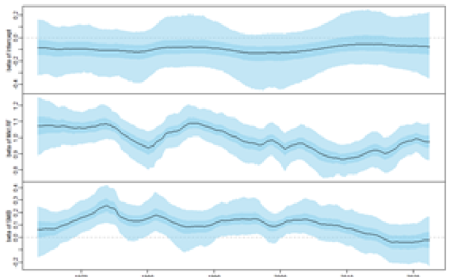

Figure 11. Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso, "Manufacturing" Sector

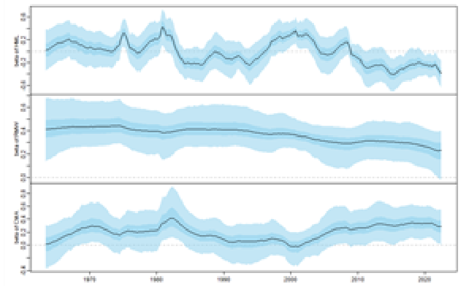

Figure 12. Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso, MAN

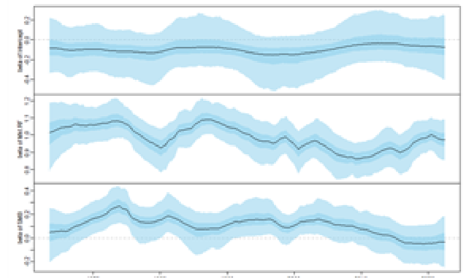

Figure 13. Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Ridge prior, MAN

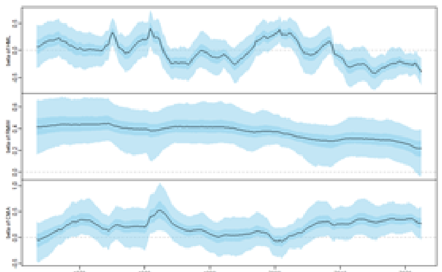

Figure 14. Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Ridge prior, MAN

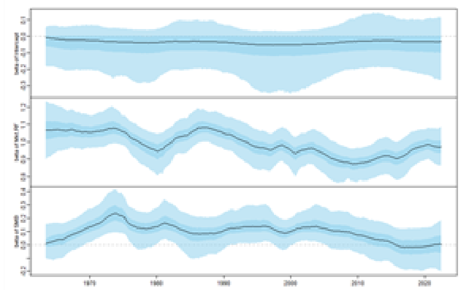

Figure 15. Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Double Gamma prior, MAN

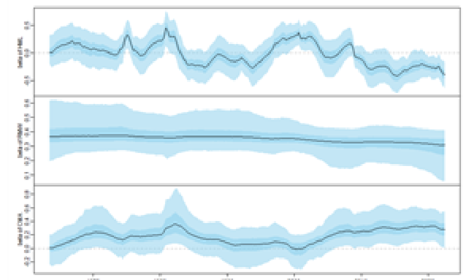

Figure 16. Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Double Gamma prior, MAN

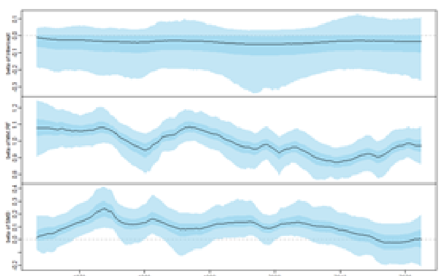

Figure 17. Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Triple Gamma prior, MAN

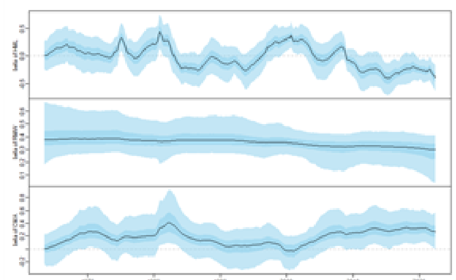

Figure 18. Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Triple Gamma prior, MAN

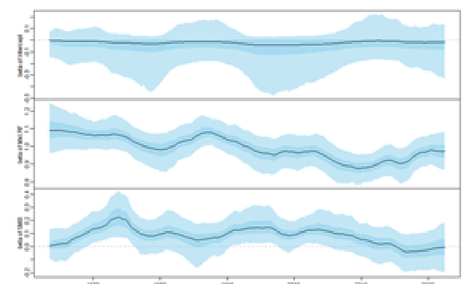

Figure 19. Time-varying effects of Intercept, SMB, and Excess Return factors Through Stochastic Volatility, MAN

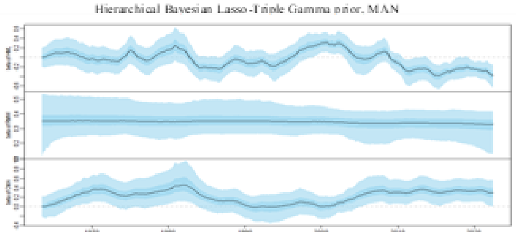

Figure 20. Time-varying effects of HML, RMW, and CMA factors Through Stochastic Volatility, MAN

Graphical representation of the results obtained, in a condensed form, of the TVPM-S methodology obtained for all the priors for the "Manufacturing" Industry.

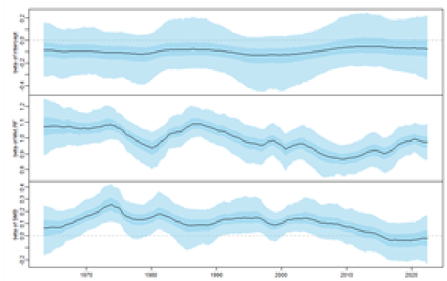

Figure 31. Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso, "Health" Sector

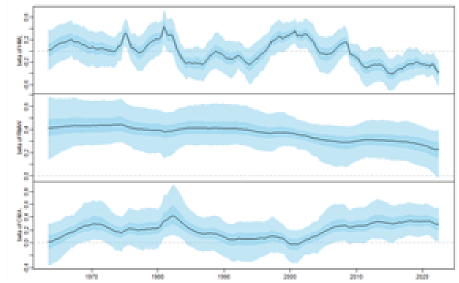

Figure 32. Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso, "Health" Sector

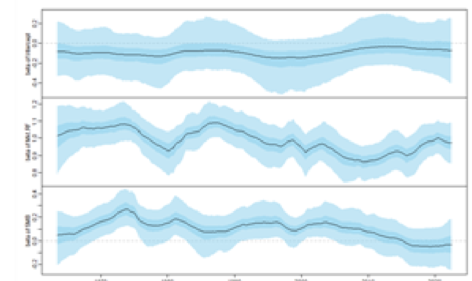

Figure 33. Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Ridge prior, HLT

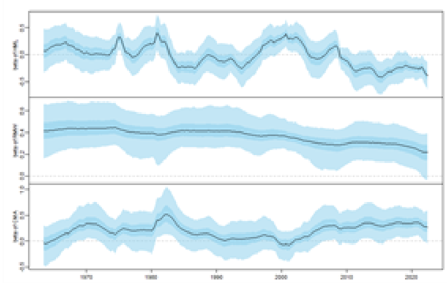

Figure 34. Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian

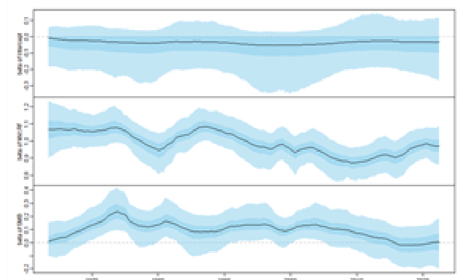

Figure 35. Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Double Gamma, HLT

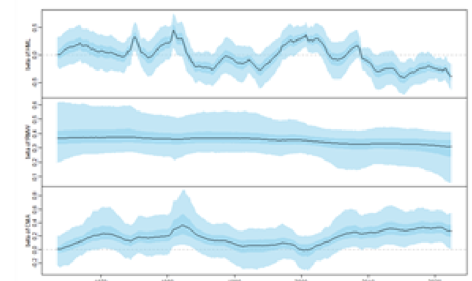

Figure 36. Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Double Gamma, HLT

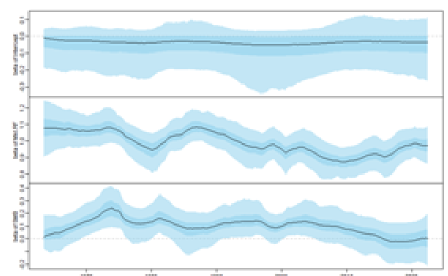

Figure 37. Time-varying effects of Intercept, SMB, and Excess Return factors for

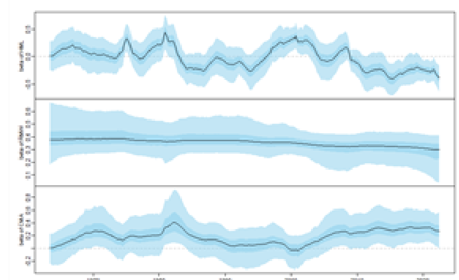

Figure 38. Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Triple Gamma, HLT

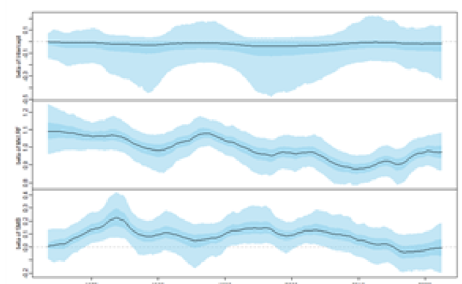

Figure 39. Time-varying effects of Intercept, SMB, and Excess Return factors Through Stochastic Volatility, HLT

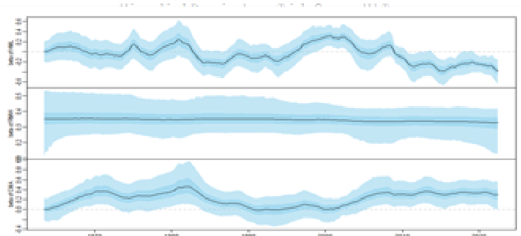

Figure 40. Time-varying effects of HML, RMW, and CMA factors Through Stochastic Volatility, HLT

Graphical representation of the results obtained, in a condensed form, of the TVPM-S methodology obtained for all the priors for the "Health" Industry.

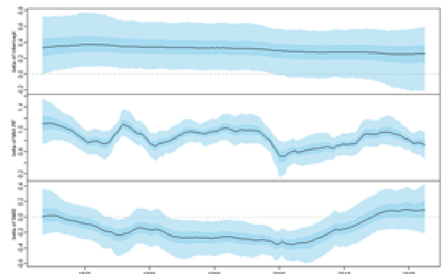

Figure 41, Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso, "Other" Sector

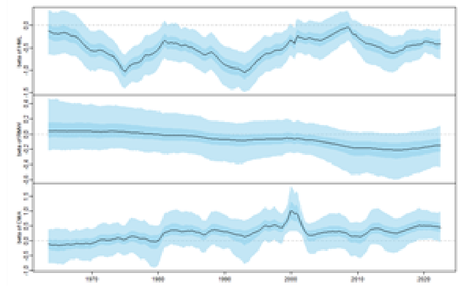

Figure 42, Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso, "Other" Sector

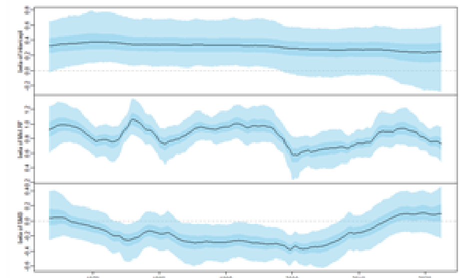

Figure 43, Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Ridge prior, OTH

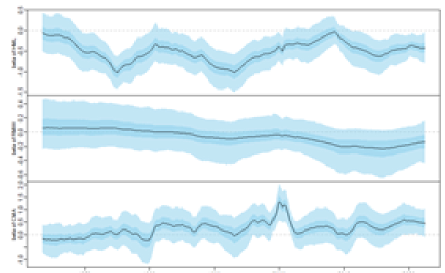

Figure 44, Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Ridge prior, OTH

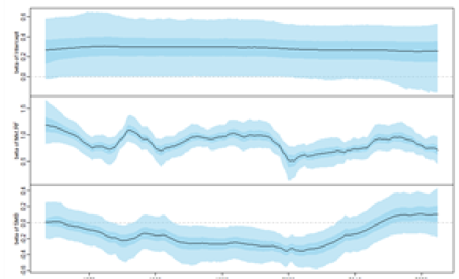

Figure 45, Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical Bayesian Lasso-Double Gamma, OTH

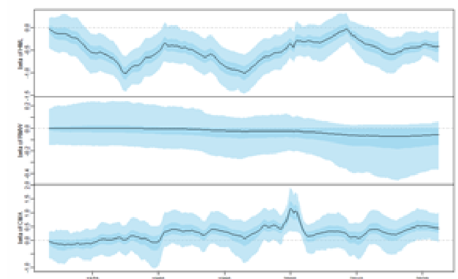

Figure 46, Time-varying effects of Intercept, HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Double Gamma, OTH

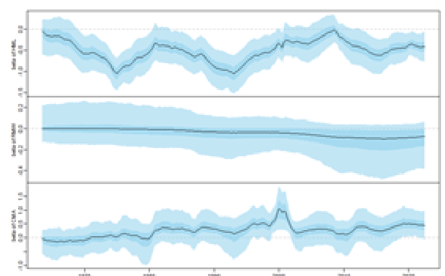

Figure 47, Time-varying effects of Intercept, SMB, and Excess Return factors for Hierarchical

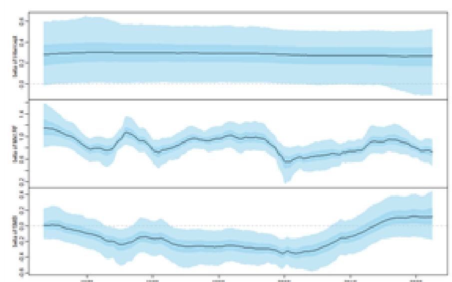

Figure 48, Time-varying effects of HML, RMW, and CMA factors for Hierarchical Bayesian Lasso-Triple Gamma, OTH

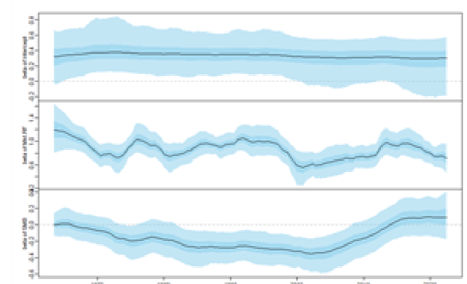

Figure 49, Time-varying effects of Intercept, SMB, and Excess Return factors Through Stochastic Volatility, OTH

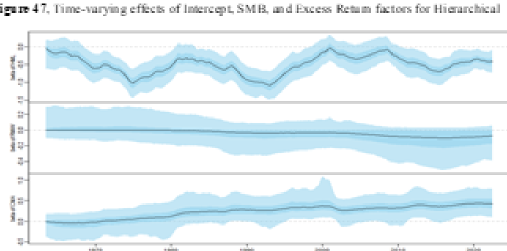

Figure 50, Time-varying effects of HML, RMW, and CMA factors Through Stochastic Volatility, OTH

Graphical representation of the results obtained, in a condensed form, of the TVPM-S methodology obtained for all the priors for the "Other" Industry.