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MODELLING AND FORECASTING NIFTY 50 USING HYBRID ARIMA-GARCH MODEL

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Abstract. This study proposes an estimation technique for developing the best fit ARIMA-GARCH model to predict the closing values of Nifty 50. The study put forward different methods to resolve the issue of non-stationarity in mean as well as variance of the series before starting the estimation process. This study has applied autoregressive integrated moving-average (ARIMA), generalized autoregressive conditional heteroscedasticity (GARCH), exponential GARCH (EGARCH) and threshold GARCH (TGARCH) model along with other estimation procedures on the daily closing prices of Nifty 50 from Jan 1, 2009 to Dec 30, 2019. Finally, the study identifies ARIMA(2,1,2)-EGARCH(1,1,1) as best model to predict the closing prices of Nifty 50. The findings indicate that the static forecast provides better results as compared to the dynamic forecast. These research findings will add to the tool kit of domestic as well as international portfolio managers and investors to frame suitable NIFTY trade strategies with least possible risks.

1. Introduction

Volatility forecasting is one of the most challenging tasks in the area of finance. It refers to make prediction about what is likely to take place in the near future after observing what has occurred formerly and what is happening at present (Idrees, Alam, and Agarwal 2019). Over the past two decades, it has gained much attention from academicians, market analysts, researchers, and consultants as forecasting plays a vital role in all the financial applications viz. portfolio management, risk management, derivatives, hedging and asset allocation (Pati, Barai, and Rajib 2017). Numerous methods and methodologies are available in the financial literature to predict the volatility based fluctuations (Ratnayaka et al. 2015). Autoregressive integrated moving average (p,d,q) is one of the most widely used statistical technique for analysing and forecasting time series data and is also known as Box and Jenkins methodology. This model is based on past values of the series along with the previous error terms for estimation (Adebiyi, Adewumi, and Ayo 2014). This model contemplates procedure for varying trend, seasonality, random noise and also for residual diagnostic and is comparatively more robust and effectual as compared to other structural models for short term forecasting (Mustapa and Ismail 2019).

Volatility models have also gained considerable attention in the financial world after the seminal Nobel Prize winning work of Engle (1982), which for the first time modeled the time-varying conditional variance with ARCH process using past disturbances of the series and allowing the variance of the error term to vary over time. For such series, recent past data give facts about one period forecast variance. But, a major limitation of this model is that it looked like a moving average than auto-regression. Hence, Bollerslev (1986) generalized the ARCH model

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by introducing a more parsimonious model that is generalized autoregressive conditional heteroskedasticity (GARCH) model, permitting more flexible lag structure. In ARCH (q) process, the conditional variance is stated as a linear function of past sample variances only, whereas the process followed in GARCH (p, q) model permits lagged conditional variances to enter in the equation as well. ARCH and GARCH are presumed to be the most conspicuous models to capture the random movement of stock prices and there have been frequent refinements to this approach to well capture the stylized features of the financial data (Karmakar 2005).

The present study will put forward an estimation technique for developing the best fit ARIMA-GARCH model to forecast the closing values of Nifty 50. Nifty 50 is National stock exchange of India's benchmark broad based stock market index. It is widely used by investors in India and around the world as a barometer of the Indian capital markets. It covers 12 sectors of the Indian economy and offers investment managers exposure to the Indian market through a single portfolio. The study has been divided into three stages to achieve its objective of economic forecasting. First stage deals with the development of an ARIMA (p,d,q) model, that best fit the movement of Nifty 50 index on the basis of statistically significant coefficients and least selected information criterion. In the second stage, symmetric as well as asymmetric GARCH family models are fit to measure the heteroskedasticity in variance of the series and also to identify the presence of leverage effect. Finally, in the third stage forecasting is done so as to judge the accuracy of the model and the comparison among the static and dynamic forecast is made.

The remaining paper is arranged as below; Section 2 presents a brief review of different prediction models presented in literature; Section 3 elaborates the time series modelling techniques and the methodology used in the present study; Section 4 demonstrates the quantitative analysis of the proposed model and the last section highlights the research findings.

2. Review of Literature

Numerous studies are available which contribute to the modelling and forecasting of the stock prices using ARIMA-GARCH model. To begin with, Henry (1998) used daily data of Hong Kong stock exchange for modelling the asymmetry of the stock market and found that GQARCH model performs to be the most appropriate model to measure the asymmetry. Similarly, Binder and Merges (2001) examined how economic forces explained the volatility of stock market using the standard deviation of S&P Composite Index and concluded that standard deviation inversely relates to the ratio of projected profits to projected revenues, implying the risk will rise during economic contractions and fall during economic recoveries. More so, Singla and Pasricha, (2012) suggested that systematic risk is linearly related to stock's return during market rises, but during falls, the relation is non-linear.

Further, Mohammadi and Su (2010) examined the behaviour of oil returns and conditional variance using weekly data of eleven global markets comprising of oil-exporting and oil- importing countries applying four types of GARCH models. The results indicated that the conditional standard deviation is more able to capture the volatility in oil returns than the traditional conditional variance. The study also considered the out-of-sample forecasting measures of four volatility models such as GARCH, EGARCH, APARCH, FIGARCH and suggested that MA(1)–APARCH(1,1) model outperforms the others. Similarly, Sopipan (2017) forecasted the volatility of gold prices using ARIMA - GARCH model using three different distributions for the innovations and advocated that ARIMA(2,0,2) gives the best performance for predicting the gold returns. The study also suggested that the cumulative returns ARIMA (2,0,2)-GARCH model with normal and GED error distribution outperforms ARIMA (2,0,2)-GARCH-t error distribution. Mustapa and Ismail (2019) also identified a suitable ARIMA model and fitted it into GARCH (1,1) model to measure the variability in variance. The results revealed that ARIMA (2,1,2)-GARCH (1,1) model is considered to be the most suited model for predicting the S&P500 stock prices and also pointed out that a dynamic forecast gives better prediction as compared to a static one.

Kumar and Dhankar (2010) investigated the asymmetric effect on volatility and stated negative significant connection among stock returns and conditional volatility. But, the association between stock yield and standardized residuals was significant. This study also highlighted the facts that investors modify their investment choices with regard to expected volatility, however, they expect additional risk premium for the unexpected volatility. One more study by Kumar and Dhankar (2011) verified the existence of non-linearity, heteroskedasticity and asymmetric nature of stock returns and reported a positive significant relationship between stock return and unexpected volatility, inferring investors anticipate more risk premium if there is any unpredicted rise or drop in stock prices.

In the same way, Xiong and Han (2015) used the Granger causality – MSV model to examine the volatility spillover and suggested that volatility spillover effects are bi-directional and asymmetric. Varughese and Mathew (2017) also showed the impact of FPI's investment on the Indian stock market volatility and opined that market is more volatile in decreasing trend rather than in increasing trend. The study results also revealed that there is a significant contribution of FPI's purchase and sales in making the stock market volatile. More so, studies conducted by Natchimuthu and Chellaswamy (2018) and Amudha and Muthukamu (2018) found the presence of volatility clustering, long term memory features and leverage effect in the NSE sectoral indices and determine that negative news have more influence on the next period volatility as asymmetrical volatility models beat the symmetrical volatility models. One of the latest studies by Kaur and Singh (2019) examined the Nifty future index and MCX composite commodity index and reported persistent volatility in both the markets, though asymmetric effect was confirmed only in stock futures market, not in the commodity future market.

Numerous scholars also equated different ARCH family models to find the best fitted model based on different information criteria. A study by Banumathy and Azhagaiah (2012) exhibited GARCH (1,1) is the best possible model to capture the symmetric volatility and for asymmetric volatility, TGARCH (1,1) to be the more suitable model. Anton (2012) suggested TGARCH and PGARCH to be the most successful models to forecast volatility. Similarly, Vasudevan and Vetrivel (2016) recommended that asymmetric models are better as compared to the symmetric GARCH model. Studies by Amudha and Muthukamu (2018); and Lama et al. (2015) also proposed that EGARCH model perform better as compared to ARIMA and GARCH models to capture the volatility.

3. Data and Methodology

The data comprises of 2657 observations of closing prices of Nifty 50 index from the period Jan. 1, 2009 to Dec. 30, 2019. The data has been retrieved from the official website of National Stock Exchange of India Limited. Further, the study used ARIMA modelling to find optimal mean equation and then added it in the symmetric as well as asymmetric GARCH models to measure the variability of the series. EViews software version 10 and MS Excel have been used for conducting the time series analysis.

3.0.1. *Identification of suitable model.* To begin with, the study observed the nature of the series by plotting the closing prices of Nifty 50 over time. The graphical presentation of data has helped to get an overview of the data and also to look at its pattern. Afterwards, Autocorrelation function (ACF) is used to examine the stationarity of the series. A series is said to be non-stationary if it's mean, variance and auto-covariances depend on the time factor and is said to be stationary if its mean, variance and auto-covariance remains the same over the entire series such that it satisfies the mean reversion criterion (Bhaumik 2015). ACF exhibits the autocorrelation and partial autocorrelation functions up to the specified order of lags. A series is considered to be stationary if ACF decays rapidly from the very first lag. On the other hand, in the case of nonstationary, the ACF dies out gradually over time. The stationarity of data is also examined using a formal test of stationarity, that is, Augmented Dickey Fuller (ADF) test. If the series is non-stationary then the transformation of the series is done to make it a stationary series. If the series follows stochastic trend, then differencing is done to make it stationary and in case the series follows deterministic trend, then detrending is done to make it a stationary series. The pattern of the correlogram, that is, ACF function and PACF function is observed to introduce the AR terms or MA terms in the ARIMA model. To identify the order of p and q in ARIMA model, trial and error method is applied. Model with significant coefficients, least information criterion values and highest R^2 is selected to get the best parsimonious model.

3.1. **Estimating the model parameters.** Firstly, ARIMA model has been applied using least square method. After getting the appropriate mean equation, it has been added in the GARCH (1, 1) model to measure the variability. Here, the study has split the data series into two parts; data from Jan 1, 2003 to Nov 30, 2019 for estimating the GARCH parameters and from Dec 1, 2019 to Dec 30, 2019 to conduct out-of-sample forecasting.

The study used symmetric as well as asymmetric GARCH models. To begin with, ARCH test is applied to examine the clustering volatility of the series. According to Engle (1982), predictable volatility depends on the past information or shocks. The ARCH model assumes that heteroskedasticity observed over different time periods is autocorrelated or it has an autoregressive structure. Afterwards, GARCH (generalized autoregressive conditional heteroskedasticity) model, introduced by Bollerslev (1986) is applied, which include the lagged conditional variance terms as autoregressive terms. The GARCH (p, q) model is written as:

$$h_t = \alpha_0 \; + \; \sum_{i=1}^{p} \alpha_i \; \varepsilon_{t-1}^2 \; + \sum_{i=1}^{q} \beta_i \; h_{t-1}^2$$

Here, the conditional variance of h at time t depends not only on the squared error term in the previous period (as in ARCH model), but, also on conditional variance of the previous period. If the conditional variance h_t is non negative, it implies that the coefficients α_0, α_i and β_i are positive numbers. The coefficient α_i (ARCH parameter) can be viewed as the news coefficient. Any increase (decrease) in the ARCH parameter signifies that news is reflected in prices promptly (gradually). It measures the effect of previous day's market price changes on today's price changes. The higher value signifies that price changes are more influenced by recent news. The coefficient β_i (GARCH parameter) can be viewed as the old news coefficient. Any increase (decrease) in the value of β_i signifies that old news has higher (lesser) persistence influence on the price changes.

The primary purpose of GARCH model is to evaluate the effects of high frequency recent information (α_i) and long-run news shocks (β_i). They are also called ARCH effect and GARCH effect respectively. The sum of α_i and β_i exhibits the volatility clustering or persistence level. It implies that large shocks persist against forecasting volatility of the following periods. As α_i + β_i approaches to unity, it indicates that persistence to the shocks of volatility has increased. Based on the volatility clustering, the trading days with high fluctuation tend to be succeeded by high volatile trading days. On the other hand, the trading days with low fluctuation tend to be followed by low volatile trading days.

One of the major restrictions of GARCH model is to give symmetric response of volatility to good and bad news. However, it has been claimed that negative shocks to financial time series probably cause more volatility than positive shocks of same magnitude (Brooks 2014). This behaviour of stock return in response to new information flow is known as asymmetric volatility (Varughese and Mathew 2017). Hence, the study employed exponential GARCH model, pioneered by Nelson (1991) using logarithmic expression of the conditional volatility in the variable, captivating the asymmetric affiliation between conditional volatility and conditional term mean. If the EGARCH asymmetry term is negative and significant, there is leverage effect, which implies that the stock market volatility is more sensitive to bad news than good ones. EGARCH model has the following specifications:

$$Logh_t^2 = \mu \ + \ \beta \log h_{t-1}^2 \ + \ \omega \left| \frac{\varepsilon_{t-1}}{\sqrt[2]{h_{t-1}^2}} \right| \ + \ \delta \left| \frac{\varepsilon_{t-1}}{\sqrt[2]{h_{t-1}^2}} \right|$$

If δ is statistically significant, there is leverage effect and it indicates that the effect of good and bad news on volatility is asymmetric. If this coefficient is negative and significant, it implies that the stock market volatility is more sensitive to bad news than good ones. The coefficient ω measures the reaction of volatility of past time into today's volatility; this is also called volatility spillover. The coefficient β measures the effects of past shocks.
The study also applied TGARCH model by Zakoian (1994), to capture the effect of good and bad news on the volatility. The model specification is as below:

$$h_t = \alpha_0 \ + \ \beta \ h_{t-1} \ + \ \alpha_1 \ \varepsilon_{t-1}^2 \ + \ \lambda \ \varepsilon_{t-1}^2 \ I_{t-1}$$

For TGARCH specification, if the asymmetric term is positive and statistically significant, it indicates that negative shocks imply a higher next period conditional variance than positive shocks of same sign. So, there would be asymmetrical impact of good and bad news.

After applying the relevant techniques, it is imperative to measure the accuracy of the applied model. To achieve this aim of the study, different diagnostic tools like Ljung-Box Q- statistic test, AR roots graph and Jarque-Bera normality test are used. If the ARMA model is correctly specified then the residuals will be white noise. It means there should be no serial correlation left in the residuals. The roots view shows the inverse roots of the AR and MA characteristics polynomial. If the ARMA process is (covariance) stationary, then all the AR roots must lie inside the unit circle. If the estimated ARMA process is invertible, then all the MA roots should lie inside the unit circle. In order to test the normality of the residuals, the Jarque-Bera test is applied. Here, the H_0 is that residuals follow normal distribution against the alternative that residuals do not obey normal distribution. However, if these conditions are not satisfied, then overfitting is done by adding more parameters in AR and MA terms. After that, the ARCH effect is verified and if it exists then the variance is heteroskedastic, which leads to the application of GARCH (1, 1) model to encounter the heterogeneity in variance.

The study has used data from Dec 1, 2019 to Dec 30, 2019 for out-of-sample forecasting with the help of dynamic and static forecasting techniques. In dynamic forecasting, previously forecasted values for the lagged dependent variables are used in forming forecasts of the current value. The static forecast calculates a sequence of one step ahead forecasts, using the actual, rather than forecasted values for lagged dependent variables. For measuring the forecast accuracy, the study used Mean Absolute Error (MAE), Root Mean Square Error (RMSE) criteria and Theil U2 coefficient (U). MAE measures the average absolute values of the differences among forecast and actual value, whereas RMSE is the square root of the average of squared errors. These values have to be least from among all possible ARIMA models that might be estimated. Theil's U statistics or Theil's coefficient of inequality provides a measure of how well a time series of estimated values compares to a corresponding time series of the observed value. Hence, it indicates better forecasting performance of the evaluated models; here, the model is considered to be satisfactory if the value of U is less than 1. The other criteria considered in the study are bias proportion, variance proportion and covariance proportion. The bias proportion states how far is the mean of the forecast from the mean of the actual series, the variance proportion conveys how far is the variance of the forecast from the variation of the actual series and the covariance proportion measures the residual unsystematic forecasting error. The forecast is considered to be good if the values of bias and variance proportion are low and that of covariance proportion is high or near to 1.

3.2. **Residual diagnostic.** After applying the relevant techniques, it is imperative to measure the accuracy of the applied model. To achieve this aim of the study, different diagnostic tools like Ljung-Box Q- statistic test, AR roots graph and Jarque-Bera normality test are used. If the ARMA model is correctly specified then the residuals will be white noise. It means there should be no serial correlation left in the residuals. The roots view shows the inverse roots of the AR and MA characteristics polynomial. If the ARMA process is (covariance) stationary, then all the AR roots must lie inside the unit circle. If the estimated ARMA process is invertible, then all the MA roots should lie inside the unit circle. In order to test the normality of the residuals, the Jarque-Bera test is applied. Here, the H_0 is that residuals follow normal distribution against the alternative that residuals do not obey normal distribution. However, if these conditions are not satisfied, then overfitting is done by adding more parameters in AR and MA terms. After that, the ARCH effect is verified and if it exists then the variance is heteroskedastic, which leads to the application of GARCH (1, 1) model to encounter the heterogeneity in variance.

3.3. **Forecasting.** The study has used data from Dec 1, 2019 to Dec 30, 2019 for out-of-sample forecasting with the help of dynamic and static forecasting techniques. In dynamic forecasting, previously forecasted values for the lagged dependent variables are used in forming forecasts of the current value. The static forecast calculates a sequence of one step ahead forecasts, using the actual, rather than forecasted values for lagged dependent variables. For measuring the forecast accuracy, the study used Mean Absolute Error (MAE), Root Mean Square Error (RMSE) criteria and Theil U2 coefficient (U). MAE measures the average absolute values of the differences among forecast and actual value, whereas RMSE is the square root of the average of squared errors. These values have to be least from among all possible ARIMA models that might be estimated. Theil's U statistics or Theil's coefficient of inequality provides a measure of how well a time series of estimated values compares to a corresponding time series of the observed value. Hence, it indicates better forecasting performance of the evaluated models; here, the model is considered to be satisfactory if the value of U is less than 1. The other criteria considered in the study are bias proportion, variance proportion and covariance proportion. The bias proportion states how far is the mean of the forecast from the mean of the actual series, the variance proportion conveys how far is the variance of the forecast from the variation of the actual series and the covariance proportion measures the residual unsystematic forecasting error. The forecast is considered to be good if the values of bias and variance proportion are low and that of covariance proportion is high or near to 1.

4. Results and Discussion

4.1. **Identification of suitable model.** The initial view about the stationarity of a series is made by plotting the series over time. It is obvious from the Figure 1 that the series drift upwards over time which means the series is non-stationary at level. It is also clear that the trend is stochastic trend so there is the presence of stochastic non-stationarity. Such a non-stationarity may be transformed into a stationary process by differencing.

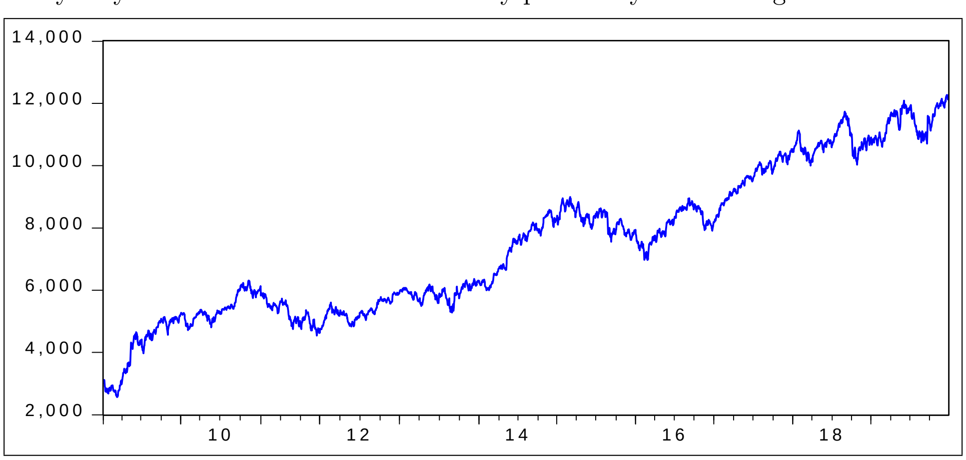

Figure 1. Daily closing prices of Nifty 50 index

It appears from Figure 2 that the ACF plot is tailing off extremely slowly in a linear way, signifying the presence of trend and non-stationarity in the series. Differencing is required to deal with such situation.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.998	0.998	2649.9	0.000
		2	0.996	-0.015	5290.6	0.000
		3	0.994	0.017	7922.2	0.000
		4	0.992	-0.004	10545.	0.000
		5	0.990	-0.016	13158.	0.000
		6	0.989	0.001	15762.	0.000
		7	0.987	0.000	18357.	0.000
		8	0.985	-0.001	20943.	0.000
		9	0.983	0.017	23520.	0.000
		10	0.981	-0.010	26088.	0.000
		11	0.979	-0.005	28647.	0.000
		12	0.977	0.008	31197.	0.000
		13	0.975	-0.000	33738.	0.000
		14	0.973	0.001	36270.	0.000
		15	0.971	-0.009	38793.	0.000
		16	0.970	0.005	41308.	0.000
		17	0.968	-0.004	43814.	0.000
		18	0.966	-0.002	46311.	0.000
		19	0.964	-0.000	48799.	0.000
		20	0.962	-0.004	51278.	0.000

Figure 2. Correlogram of Nifty 50 index

It is obvious from Figure 3 that after differencing the series show no tendency to drift upward over time, hence the series is mean stationary. But, as the time passes, the gap between the peaks and troughs increase, so the series is non-stationary in variance.

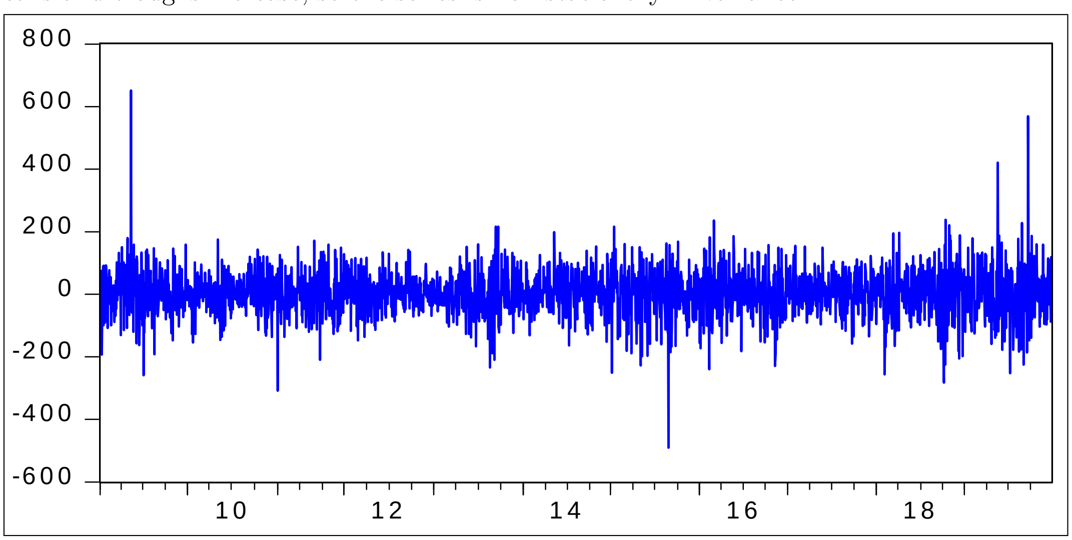

Figure 3. Nifty 50 index after first differencing

A more formal test of stationarity is the ADF unit root test. The results depicted in Table 1 revealed that the series has a unit root. Hence, it is non-stationary at level. On examining at first difference, it is observed that the series is difference stationary or it is I(1). Next, the model parameters are estimated.

Table 1. ADF Unit Root Testing			
Model	Test for unit root at	t-statistics	Prob.
Trend and intercept	Level	-2.945037	0.1484
	First difference	-48.47444	0.0000

4.2. **Estimating the model parameters.** It is obvious from Figure 2 that the ACF remains large at all the lags and the PACF cuts after the first lag. Hence, the study firstly used the simplest models; AR(1), MA(1) and ARIMA(1,1,1). Here, the trial and error method has been used until a model with all the significant coefficients, least information criterions and highest adjusted R^2 is obtained. Table 2 describes different models with their diagnosis and clearly reveals that ARMA (2, 2) is having significant coefficients, least information criterion (AIC) and highest adjusted R^2. It is important to mention that the study has used first difference price series to compare different models.

Table 2. Comparison of Different Models					
Model	Coefficients*	AIC	SIC	HQC	Adj. RÂ^2
AR(1)	Significant	11.40640	11.41305	11.40881	0.002902
MA(1)	Significant	11.40633	**11.41298**	**11.40873**	0.002974
ARMA(1,1)	Not significant	11.40706	11.41592	11.41026	0.002623
AR(2)	Not significant	11.40698	11.41585	11.41019	0.002695
MA(2)	Not significant	11.40704	11.41590	11.41025	0.002642
ARMA(2,1)	Significant	11.40648	11.41756	11.41049	0.003573
ARMA(1,2)	Significant	11.40666	11.41774	11.41067	0.003391
ARMA(2,2)	Significant	**11.40624**	11.41953	11.41105	**0.004193**
AR(3)	Not significant	11.40720	11.41828	11.41121	0.002853
MA(3)	Not significant	11.40721	11.41829	11.41122	0.002846
ARMA(3,1)	Not significant	11.40696	11.42026	11.41177	0.003469
ARMA(1,3)	Not significant	11.40703	11.42033	11.41185	0.003398
*Here, coefficients represent all the coefficients in that particular model.					

After trial and error process, ARIMA (2,1,2) is considered to be the significant model. Here, it is crucial to note that MA (1) model is also having significant coefficients, lowest SIC and HQC criterions, but after considering the value of adjusted R^2, ARIMA (2,1,2) is the most appropriate model as depicted in Table 3.

Table 3. ARIMA (2,1,2) model for Nifty 50 closing prices				
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	3.477435	1.407761	2.470189	0.0136
AR(1)	1.386764	0.100154	13.84638	0.0000
AR(2)	-0.752803	0.100338	-7.502662	0.0000
MA(1)	-1.341983	0.108070	-12.41768	0.0000
MA(2)	0.701844	0.108605	6.462355	0.0000
R-squared	0.006068	Mean dependent var	3.472289	
Adjusted R-squared	0.004193	S.D. dependent var	72.61344	
S.E. of regression	72.46106	Akaike info criterion	11.40624	
Sum squared resid	13914106	Schwarz criterion	11.41953	
Log likelihood	-15141.49	Hannan-Quinn criterion.	11.41105	
F-statistic	3.235633	Durbin-Watson stat	1.973175	
Prob(F-statistic)	0.006461	Mean dependent var	3.472289	

It is obvious from the Table 3 that all the coefficients are significant as the p-value is less than 0.05. The value of AIC is lowest and adjusted R^2 is highest as compared to the other models. Therefore, the model is considered to be desirable which can be further diagnosed with the help of residual diagnosis.

4.3. **Residual Diagnosis.** Figure 4 exhibits the correlogram of residuals of the ARIMA (2,1,2) model which reveals that there are no significant spikes outside the confidence interval. Hence, there is no autocorrelation in the series and residuals are independently distributed.

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.013	0.013	0.4781	
		2	-0.022	-0.022	1.7584	
		3	-0.005	-0.004	1.8134	
		4	0.021	0.021	3.0247	
		5	0.007	0.006	3.1551	0.076
		6	-0.016	-0.015	3.8255	0.148
		7	0.008	0.009	4.0156	0.260
		8	-0.039	-0.040	8.0254	0.091
		9	0.021	0.023	9.2528	0.099
		10	0.019	0.018	10.243	0.115
		11	-0.007	-0.007	10.358	0.169
		12	-0.003	-0.000	10.381	0.239
		13	-0.011	-0.011	10.692	0.297
		14	0.016	0.014	11.368	0.330
		15	-0.024	-0.024	12.933	0.298
		16	0.010	0.010	13.221	0.353
		17	0.018	0.019	14.135	0.364
		18	-0.003	-0.003	14.157	0.438
		19	0.001	0.002	14.162	0.513
		20	-0.007	-0.007	14.290	0.577

Figure 4. Ljung-Box test for ARIMA(2,1,2) model

Model adequacy can also be checked with the help of roots graph. The ARMA process is invertible if AR roots and MA roots lie inside the unit circle. Figure 5 shows that all the AR roots and MA roots are laying inside the unit circle. So, the model is adequate and forecasting is possible with this model.

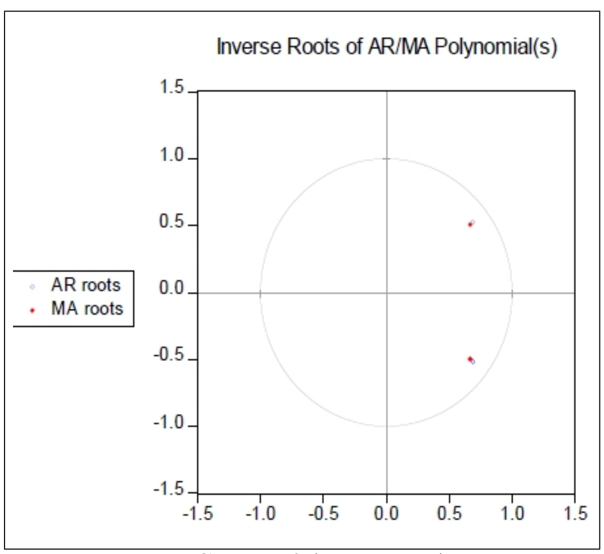

Figure 5. Graph of AR and MA Roots

It is observed from Figure 6 that the p-value of Jarque-Bera test is less than 0.05, so the residuals are not normally distributed. Hence, the study will prefer to use Generalized Error

Distribution (GED) instead of Normal (Gaussian) distribution in the estimation of GARCH model.

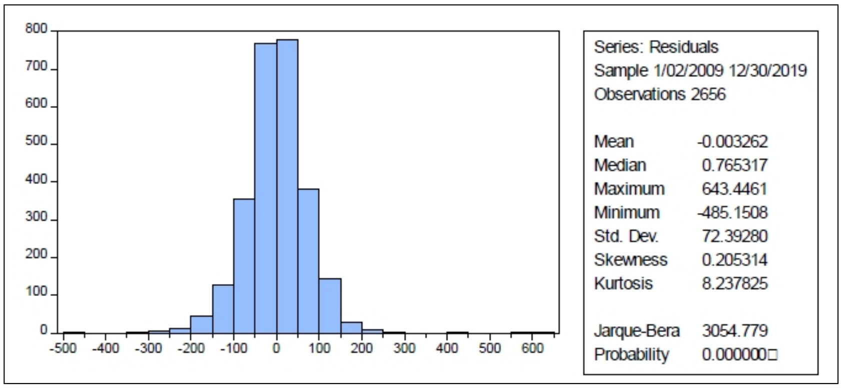

Figure 6. Histogram of the residuals

The heteroscedasticity in variance is confirmed with the help of ARCH-LM test before fitting the GARCH (1,1) into ARIMA (2,1,2) model. The results are depicted in Table 4; the p-value is 0.0000, which is significant at 5% level and confirms the existence of ARCH effect. It means volatility clustering is confirmed. Therefore, symmetric as well as asymmetric ARCH family models like GARCH, TARCH and EGARCH can be applied.

Table 4. Heteroskedasticity Test

F-statistic	19.63063	Prob. F(1,2653)	0.0000
Obs*R-squared	19.50113	Prob. Chi-Square(1)	0.0000

Table 5 depicts the estimates of ARIMA (2,1,2)-GARCH, TGARCH, EGARCH model. The estimates of GARCH (1,1) model for closing prices of Nifty 50 show that all the parameters are positive, thus indicating that the model is well specified to understand the volatility of nifty returns. The coefficient α (ARCH parameter) can be viewed as news coefficient. The positive and significant value indicates that news about previous volatility (past squared residual) has an explanatory power on current volatility. The coefficient of β (GARCH parameter) can be viewed as an old news coefficient. The positive and significant value indicates that past volatility has a persistence effect on current volatility. The sum of ARCH and GARCH coefficients ($\alpha + \beta$) is closer to unity (0.985521) which indicates that there is significant persistence in volatility, thus implying that large changes in prices are likely to be followed by large changes and small changes in prices are likely to be followed by small changes.

Table 5. Parameters' Estimates of different ARCH Family Models

Conditional volatility model	C	ARCH (-1)(α)	GARCH (-1)(β)	Leverage effects(γ_1)	AIC	SBC	HQC
GARCH	80.11521	0.053001	0.932520	-	11.27904	11.29912	11.28631
	(0.0071)	(0.0000)	(0.0000)				
TGARCH	165.3174	0.010241	0.901739	0.114627	11.26640	11.28871	11.27448
	(0.0001)	(0.2514)	(0.0000)	(0.0000)			
EGARCH	0.176211	0.125240	0.967737	-0.086140	11.26082	11.28313	11.26890
	(0.0052)	(0.0000)	(0.0000)	(0.0000)			

*values in parenthesis are the p-values.

The asymmetry terms are positive and highly significant in case of TGARCH specification, but, negative and highly significant for EGARCH specification which confirms the existence of leverage effect in the Indian stock market and implies that bad or negative news has a greater impact on market volatility than good or positive news. The same result is also confirmed in a study conducted by Chuliá, Martens and Dijk, (2010), which proposed that average response to negative shocks (bad news for stocks) is larger than positive shocks (good news for stocks).

In order to select the best model from among the different GARCH family models, the values of different coefficients and different information criteria under different versions are compared. It is apparent that the ARIMA(2,1,2)-EGARCH(1,1,1) is the best fit model for Nifty 50 index prices as the model is having significant coefficients, least AIC (11.26082), SIC (11.28313) and HQC (11.26890) values. Moreover, these values are also smaller than that the values under ARIMA(2,1,2) model.

4.4. **Forecasting Nifty 50 using the ARIMA(2,1,2)-EGARCH(1,1,1) model.** .
The ARIMA(2,1,2) - EGARCH(1,1,1) model is estimated using observations from Jan 1, 2009 to Nov 30, 2019 (2637 observations) and left over observations from Dec 1, 2019 to Dec 30, 2019 are used to conduct out of sample forecasts for Nifty 50 closing prices. The results of static and dynamic forecast are depicted in Table 6. The values of bias and variance proportion are low and that of covariance proportion is high, hence the forecasts may be considered satisfactory. Further, the values of RMSE and MAE are lower in case of static forecast as compared to the dynamic forecast and Theil U2 coefficient (U) is approximate to 1, which is satisfactory. Hence, the study concludes that static forecast gives a better prediction of nifty 50 future prices as compared to the dynamic forecast.

Table 6. The Forecast results on the basis Dynamic Forecast and Static Forecast

Forecast: Dynamic Forecast		Forecast: Dynamic Forecast	
Actual: Price		Actual: Price	
Forecast sample: 12/02/2019 to 12/30/2019		Forecast sample: 12/02/2019 to 12/30/2019	
Included observations: 20		Included observations: 20	
Root Mean Squared Error	118.3482	Root Mean Squared Error	64.72328
Mean Absolute Error	100.3333	Mean Absolute Error	53.62288
Mean Abs. Percent Error	0.829579	Mean Abs. Percent Error	0.444123
Theil Inequality Coefficient	0.004894	Theil Inequality Coefficient	0.002677
Bias Proportion	0.003243	Bias Proportion	0.009831
Theil U2 Coefficient	1.815034	Theil U2 Coefficient	0.994866
Symmetric MAPE	0.829696	Symmetric MAPE	0.444397

Afterwards, the stock prices for the month of Dec, 2019 have been forecasted with the help of dynamic and static forecast. Table 7 (bext page) depicts the actual value of Nifty 50 closing prices, the forecast prices and the difference between the actual and forecast values for the month Dec, 2019. It is apparent that the difference is least in case of static forecast, hence it is considered to be a satisfactory forecast.

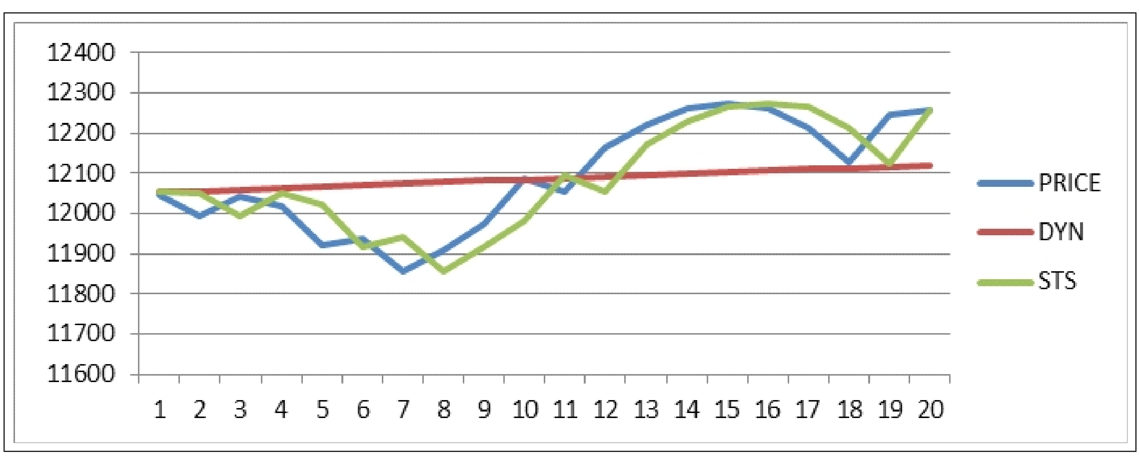

Figure 7. Comparison between the observed and the forecasted stock prices

The same thing is also depicted graphically in Figure 7, here, on Y axis, there are the actual values of Nifty 50 closing prices with the forecasted values of Nifty 50 using static and dynamic forecast for the month of Dec, 2019, signifying the fact that the static forecast moves in line with the actual series, whereas the pattern of dynamic model is directional. The results of RMSE, MAE and TIC also exhibit similar results.

Table 7. Comparison between the observed and the forecasted stock prices

Date	Observation (O)	Dynamic forecast value, (D)	Static forecast value, (S)	Difference = O - D	Difference = O - D
12-02-2019	12048.2	12053.01	12053.01	-4.80769	-4.80769
12-03-2019	11994.2	12055.06	12049.94	-60.8586	-55.739
12-04-2019	12043.2	12060.02	11995.51	-16.8213	47.68641
12-05-2019	12018.4	12063.97	12049.93	-45.5668	-31.5291
12-06-2019	11921.5	12067.2	12020.73	-145.7	-99.2278
12-09-2019	11937.5	12070.63	11917.7	-133.131	19.80012
12-10-2019	11856.8	12074.23	11941.88	-217.433	-85.0783
12-11-2019	11910.15	12077.8	11856.6	-167.648	53.54924
12-12-2019	11971.8	12081.32	11916.48	-109.523	55.3218
12/13/2019	12086.7	12084.85	11980.11	1.846314	106.5912
12/16/2019	12053.95	12088.39	12096.95	-34.4442	-43.0029
12/17/2019	12165	12091.93	12054.33	73.06616	110.6714
12/18/2019	12221.65	12095.47	12173.88	126.1788	47.77033
12/19/2019	12259.7	12099.01	12229.67	160.6913	30.02968
12/20/2019	12271.8	12102.55	12264.36	169.2533	7.436532
12/23/2019	12262.75	12106.08	12274.97	156.6653	-12.2151
12/24/2019	12214.55	12109.62	12265.25	104.9275	-50.6984
12/26/2019	12126.55	12113.16	12214.79	13.38959	-88.2382
12/27/2019	12245.8	12116.7	12124.25	129.1017	121.5463
12/30/2019	12255.85	12120.24	12257.37	135.6138	-1.51824

5. Conclusion

The present study suggest the estimation process for developing the best fitted ARIMA-GARCH model to forecast the values of Nifty 50 closing prices. Numerous methods are put forward to solve the non-stationarity in mean and variance of the series before beginning the estimation process. Then, ARIMA modelling is used to find optimal mean equation, which is further added in the GARCH model to measure the variability of the series. In order to capture the leverage effect, EGARCH and TGARCH models are applied. For selecting the best model from among the different GARCH models, the values of different coefficients and information criteria under different versions are compared and ARIMA(2,1,2)-EGARCH(1,1,1) is selected as the best fit model for forecasting Nifty 50 stock prices. Finally, the empirical results reveal that static forecast gives better prediction of Nifty 50 closing prices as compared to the dynamic forecast.

However, this study was restricted only to short-term forecasts. For further studies, other volatility models such as GJR-GARCH or QGARCH can be considered with Markov regime switching model. Researchers can also use Hybrid ANN-ARIMA model to achieve better forecast. Finally, it is supposed that present study will definitely provide good insights to the investors and portfolio managers and would help them to make better portfolio decision.

References

[1] Adebiyi, Ayodele Ariyo, Aderemi Oluyinka Adewumi, and Charles Korede Ayo. 2014. "Comparison of ARIMA and Artificial Neural Networks Models for Stock Price Prediction." Journal of Applied Mathematics, 1–7. https://doi.org/dx.doi.org/10.1155/2014/614342.

[2] Ayub, Shafaq, and Yasmin Zahra Jafri. 2020. "Comparative Study of an ANN-ARIMA Hybrid Model for Predicting Karachi Stock Price." American Journal of Mathematics and Statistics 10 (1): 1–9. https://doi.org/10.5923/j.ajms.20201001.01.

[3] Banumathy, Karunanithy, and Ramachandran Azhagaiah. 2012. "Modelling Stock Market Volatility: Evidence from India." Managing Global Transitions 13 (1): 27–42.

[4] Bhaumik, Sankar Kumar. 2015. Principles of Econometrics. First edit. United Kingdom: Oxford University Press.

[5] Binder, John J, and Matthias J Merges. 2001. "Stock Market Volatility and Economic Factors." Review of Quantitative Finance and Accounting 17: 5–26.

[6] Bollerslev, Tim. 1986. "Generalized Autoregressive Conditional Heteroskedasticity." Journal of Econometrics 31: 307–27.

[7] Brooks, Chris. 2014. Introductory Econometrics for Finance. New York: Cambridge University Press.

[8] Chuliá, Helena, Martin Martens, and Dick Van Dijk. 2010. "Asymmetric Effects of Federal Funds Target Rate Changes on S & P100 Stock Returns , Volatilities and Correlations." Journal of Banking and Finance 34 (4): 834–39. https://doi.org/10.1016/j.jbankfin.2009.09.012.

[9] Engle, Robert F. 1982. "Autoregressive Conditional Heteroskedasticity with Estimates of the Variance of United Kingdom Inflation." Econometrica 50 (4): 987–1008.

[10] Henry, Olan. 1998. "Modelling the Asymmetry of Stock Market Volatility." Applied Financial Economics 8: 145–53. https://doi.org/10.1080/096031098333122.

[11] Idrees, Sheikh Mohammad, M Afshar Alam, and Parul Agarwal. 2019. "A Prediction Approach for Stock Market Volatility Based on Time Series Data." IEEE Access 20: 1–9. https://doi.org/10.1109/ACCESS.2019.2895252.

[12] Karanasos, Author M, and J Kim. 2003. "Moments of the ARMA — EGARCH Model." The Econometrics Journal 6 (1): 146–66.

[13] Karmakar, Madhusudan. 2005. "Modeling Conditional Volatility of the Indian Stock Markets." Vikalpa 30 (3): 21–37.

[14] Kaur, Harpreet, and Ravinderjit Singh. 2019. "Modelling Volatility Clustering and Asymmetry: A Study of Indian Index Futures Markets." Indian Journal of Finance 13 (3): 51–65.

[15] Kumar, Rakesh, and Raj S Dhankar. 2010. "Empirical Analysis of Conditional Heteroskedasticity in Time Series of Stock Returns and Asymmetric Effect on Volatility." Global Business Review 11 (1): 21–33. https://doi.org/10.1177/097215090901100102.

[16] Kumar, Rakesh, and Raj S Dhankar. 2011. "Non Linearity and Heteroskedasticity Effect on Stock Returns Volatility: A Case of US Stock Market." Global Business Review 12 (2): 319–29. https://doi.org/10.1177/097215091101200209.

[17] Lama, Achal, Girish K Jha, Ranjit K Paul, and Bishal Gurung. 2015. "Modelling and Forecasting of Price Volatility: An Application of GARCH and EGARCH Models." Agricultural Economics Research Review 28 (1): 73–82. https://doi.org/10.5958/0974-0279.2015.00005.1.

[18] Mohammadi, Hassan, and Lixian Su. 2010. "International Evidence on Crude Oil Price Dynamics: Applications of ARIMA-GARCH Models." Energy Economics 32: 1001–8. https://doi.org/10.1016/j.eneco.2010.04.009.

[19] Mustapa, Farah Hayati, and Mohd Tahir Ismail. 2019. "Modelling and Forecasting S & P 500 Stock Prices Using Hybrid Arima-Garch Model." Journal of Physics: Conference Series 1366: 1–13. https://doi.org/10.1088/1742-6596/1366/1/012130.

[20] Natchimuthu, Nm, and Karthigai Prakasam. 2018. "Estimation of Conditional Volatility Models for Indian Stock Market." Asian Journal of Research in Business Economics and Management 8 (5): 33–48. https://doi.org/10.5958/2249-7307.2018.00045.2.

[21] Nelson, Daniel B. 1991. "Conditional Heteroskedasticity in Asset Returns: A New Approach." Econometrica 59 (2): 347–70.

[22] Pati, Pratap Chandra, Parama Barai, and Prabina Rajib. 2017. "Forecasting Stock Market Volatility and Information Content of Implied Volatility Index." Applied Economics, 1–17. https://doi.org/10.1080/00036846.2017.1403557.

[23] Ratnayaka, R.M. Kapila Taranga, D.M.K.N. Seneviratne, Wei Jianguo, and Hasitha Indika Arumawadu. 2015. "A Hybrid Statistical Approach for Stock Market Forecasting Based on Artificial Neural Network and ARIMA Time Series Models." International Conference on Behavioral, Economic, and Socio-Cultural Computing, 54–60.

[24] Singla, Ravi, and J.S. Pasricha. 2012. "Asset Pricing in the Indian Capital Market: A Study of Positive and Negative Return Periods." Journal of Academic Research in Economics 4 (1): 90–101.

[25] Sopipan, Nop. 2017. "Trading Gold Future with ARIMA-GARCH Model." Thai Journal of Mathematics, no. June 2017: 227–38. https://doi.org/http://thaijmath.in.cmu.ac.th.

[26] Sorin, Anton. 2012. "Evaluating the Forecasting Performance of GARCH Models . Evidence from Romania." Procedia - Social and Behavoiural Sciences 62: 1006–10. https://doi.org/10.1016/j.sbspro.2012.09.171.

[27] Varughese, Ashish, and Tomy Mathew. 2017. "Asymmetric Volatility of the Indian Stock Market and Foreign Portfolio Investments: An Empirical Study." Indian Journal of Finance 11 (6): 36–49.

[28] Vasudevan, R. D., and S. C. Vetrivel. 2016. "Forecasting Stock Market Volatility Using GARCH Models: Evidence from the Indian Stock Market." Asian Journal of Research in Social Science and Humanities 6 (8): 1565–74. https://doi.org/10.5958/2249-7315.2016.00694.8.

[29] Xiong, Zhengde, and Lijun Han. 2015. "Volatility Spillover Effect between Financial Markets: Evidence since the Reform of the RMB Exchange Rate Mechanism." Financial Innovation 1 (9): 1–12. https://doi.org/10.1186/s40854-015-0009-2.

[30] Zakoian, Jean Michel. 1994. "Threshold Heteroskedastic Models." Journal of Economics Dynamics and Control 18: 931–55.