The Review of Finance and Banking
Volume 02, Issue 2, Year 2010, Pages 083–94
S print ISSN 2067-2713, online ISSN 2067-3825

HAVE VOLATILITY SPILLOVER EFFECTS OF COINTEGRATED EUROPEAN STOCK MARKETS INCREASED OVER TIME?

KLAUS GROBYS

Abstract. In this study volatility spillover effects in preselected cointegrated European stock markets are investigated. The data generating processes are estimated by applying Vector-Auto Regression (VAR) models. Thereby, the impacts of volatility spillovers are measured by a new concept being denoted here as Volatility Impulse Response Density Functions (VIRDF) being an advancement of the Volatility Impulse Response Functions (VIRF) methodology. A sample-split analysis covering daily data from 26.11.1990-05.10.2000 and 06.10.2000-28.05.2010 reveals that the volatility spillover impact from the German stock market to the Swedish and British stock markets have increased by 73.87%, respectively, 15.52%.

1. Introduction

The studies of the transmission of shocks in financial markets across economies have become an important issue in the international financial literature. Eun and Shim (1989) and Becker et al. (1990) investigate such spillover effects for instance. Thereby, their models involve Vector Autoregressive (VAR) models or a set of single linear equations attempting to capture the dependencies between international equity returns. These contributions though are focused on the asset return series and on the question how returns are correlated across different economies. Consequently, they consider only interdependence through the mean of the stochastic process. A recent study of Gklezakou and Mylonakis (2010) support the previous finding that correlations between different economies' stock markets have empirically increased over time.

Apart from the concept of correlation, other studies are focused on searching for cointegration relationships between stock markets in order to figure out interdependencies between these markets. Corhay et al. (1993), for instance, finds cointegration relationships among stock prices in different European countries except for Italy. Arshanapalli and Doukas (1993) find interdependencies between US and European markets, like the British, German, and French stock markets using the bivariate Engle and Granger (1987) methodology. Furthermore, Pascual (2003) figures out evidence of increasing financial integration for France. His result is confirmed by the presence of significant trends in the responsiveness of the French stock market to the level of the British, German, and French stock prices.

Other contributions focus explicitly on the volatility of asset returns, suggesting the existence of second order dependencies. In this context, Hamao et al. (1990) employs univariate generalized autoregressive conditional heteroskedastic (GARCH) models to figure out the dynamic behaviors of international stock markets against the background of the 1987 US-stock market crash. They find evidence of significant price-volatility spillover effects arising from the

Received by the editors July 25, 2010. Accepted by the editors December 01, 2010.
Keywords: volatility spillovers, volatility impulse response functions, stock market, GARCH, BEKK.
JEL Classification: G15, C22, C32.
Klaus Grobys works in the Merchant Control Department of KLARNA AB, Stockholm.
Email: klausgrobys@hotmail.de.
This paper is in final form and no version of it will be submitted for publication elsewhere.

US-stock market passing over the British and Japanese markets and from the British market merging to the Japanese market with respect to the post-crash period.

Panopoulou and Pantelidis (2005) investigate the international information transmission between the U.S. and the rest of the G-7 countries employing daily stock market return data covering 20 years (i.e. from 31.12.1985 to 08.10.2004). Their sample-split analysis gives evidence that the linkages between the markets have substantially changed in the more recent era (i.e. post-1995 period), suggesting increased interdependence in the volatility of the markets under consideration.

In contrast to most of the other studies focusing on spillover effects stemming from the US-market, in the following work though European stock markets are considered only. On Thursday, the 20.05.2010, the Swedish stock index OMX 30 decreased by 2.30% even though there was no evidence given of a significant change (i.e. extraordinary bad news) at the Swedish stock market. The Swedish newspaper Dagens Nyheter reported the day afterwards that more rigorous rules concerning short sales in Germany stood behind this response.[1] On the 20.05.2010 a new law was adopted in Germany that short sales of some financial stocks are forbidden prospectively. But have these spillover effects increased over time in general? In the following contribution, a model will be suggested which exhibits both, adequate estimates concerning the stock markets' mean processes and a method to determine changes concerning second-order moments over time. The model being based on VAR and a refinement of multivariate GARCH models shows that volatility spillover impacts from the German to the Swedish and British stock markets increased by 73.87%, respectively, 15.52% when comparing the last two decades.

2. Literature Review

Traditional impulse response analysis is applied for orthogonal transformations of the disturbance vector. These transformations involve a causality scheme which researchers have to assume a priori. Assuming pre-specified causality patterns is often used when analyzing macroeconomic systems, for instance. Hafner and Herwartz (2001) mention, however, that it appears hardly feasible to impose realistic causality structures a priori in the context of financial time series, as they are typically highly interrelated and observed at high frequencies. News is via definition inherently uncorrelated over time. If there were any predictable patterns of news, Hafner and Herwartz (2001) argue that hedge portfolios could be constructed which would at least partially eliminate the risks being associated. However, this would contradict the assumption of news being unsystematic risk and unhedgeable. Therefore, Hafner and Herwartz (2001, 2006) define news to be risk sources that are independent over time and thus unpredictable.

Panopoulou and Pantelidis (2005) mention that the concept of volatility impulse response functions (VIRF) as introduced by Hafner and Herwartz (2001, 2006) accounts for the following: First, this method allows researchers to determine exactly how a shock to one market influences the dynamic adjustment of volatility to another market as well as the persistence of these spillover effects. Second, VIRFs depend on both, the state of the actual system's volatility and the unexpected returns vector, ξ_0, denoting the initial volatility shock. The third point is that in contrast to typical impulse response functions, this specific method avoids typical orthogonalization as well as ordering problems which would be hardly feasible in the presence of highly interrelated financial time series.

The following methodology is organized as follows: First, the data generating mean process of preselected European stock markets will be modeled. Thereby, the concept of cointegration matters. Whether time series are cointegrated, the spurious regression problem can be neglected. The presence of a cointegration relationship between financial time series gives reasons for causality per definition. Moreover, estimating a model involving cointegrated assets via OLS exhibits parameter estimates that are in line with Stock (1987) super-consistent. Since modeling

[1]See *Dagens Nyheter*, (Ekonomi) 21.Mai 2010, p.5.

the second-order moment is based on estimated disturbances, the property of super-consistency may exhibit more accurate estimates concerning spillover effects.

Thereby, the concept of volatility impulse response functions is refined to the concept of volatility impulse response density functions being able to capture the overall volatility impact from one stock market to another. As volatility shocks are continuous random variables, the probability of such shocks have to be taken into account, too. Thus, volatility impulse response density functions give a precise estimation of the overall volatility spillover effects occurring within a certain time window. As a consequence, increasing or decreasing volatility spillover effects may be estimated as the difference of the volatility spillover effects between different time windows.

3. Statistical Model

In line with Johansen (1988, 1991), Johansen and Juselius (1990, 1991) the multivariate trace test for cointegration is employed to test whether the stock indices have a cointegration relationship. The Johansen procedure employs the maximum likelihood estimates of a fully specified error correction model which is given by

$$\Delta Y_t = \mu \sum_{i=1}^{k} \Gamma_i \Delta Y_{t-1} \Pi Y_{t-1} + \varepsilon_t \tag{3.1}$$

where ΔY_t exhibits the vector of the stock markets' price changes in logarithms at time t, μ is a constant vector, Γ represent the short-run impact, and Π denotes the long-run impact matrix having reduced rank under cointegration. If rank of Π is equal to one, the stock indices will be cointegrated. To determine the rank r of the estimated long-run matrix $\widehat{\Pi}$, the eigenvalues $\widetilde{\lambda}_i$ have to be calculated. Thereby, the number of significantly nonzero eigenvalues shows the rank of $\widehat{\Pi}$, and can be evaluated by the trace test. Then, the trace test statistic is the result of testing the restriction $r \leq q(q < n)$ against the completely unrestricted model $r \leq n$:

$$\lambda_{trace} = -T \sum_{i=q+1}^{n} \ln(1 - \widetilde{\lambda}_i) \tag{3.2}$$

where T is the sample size and $\widetilde{\lambda}_{r+i}, ..., \widetilde{\lambda}_n$ are in line with Kaiser and Füss (2007) the $n - r$ smallest squared canonical correlations. Stock (1987) has shown that OLS estimators will be super consistent if stochastic processes are cointegrated. Therefore, the price processes of the stock indices can be modeled via Vector Auto-Regression (VAR) of lag-order L which is given by

$$Y_t = \upsilon_0 + A_t Y_{t-1} + ... + A_L Y_{t-L} + \varepsilon_t \tag{3.3}$$

where Y_t exhibits the stock markets' prices in logarithms where υ_0 is a constant vector, $A_1, ..., A_L$ are parameter matrices, and $\varepsilon_t \sim (0, \Sigma_0)$. The covariance matrix Σ_0 exhibits the unconditional variances and covariances concerning the whole sample. To determine whether the volatility spillover effects have changed over time, the whole sample of sample size T is divided into two subsamples of equal length. Moreover, the covariance matrices $\Sigma_{1,t}$ and $\Sigma_{2,t}$ of the first, respectively, second sample are in line with Hafner and Herwartz (2001, 2006) allowed to vary over time $t = 1, ..., T_1$ (i.e. the first subsample) and $t = T_1 + 1, ..., T_2$ (i.e. the second subsample). Thereby, it is assumed that $\Sigma_{1,t}$ and $\Sigma_{2,t}$ follow a bivariate GARCH(1,1) model given by

$$vech(\Sigma_t) = c + \sum_{i=1}^{p} A_i vech(u_{t-i} u'_{t-i}) + \sum_{i=1}^{q} B_i vech(\Sigma_{t-i}) \tag{3.4}$$

where $q = p = 1$. The vech-operator denoted by $vech(.)$, stacks the lower fraction of a quadratic matrix of the dimension N into a vector $N(N + 1)/2$. Hafner and Herwartz (2001, 2006) and Panopoulou and Pantelidis (2005) suggest employing the BEKK representation as introduced by Engle and Kroner (1995) which is given by

$$\Sigma_t = C_0' C_0 + \sum_{k=1}^{K}\sum_{i=1}^{p} D_{ki}' \varepsilon_{t-1}\varepsilon_{t-1}' D_{ki} + \sum_{k=1}^{K}\sum_{i=1}^{p} G_{ki}' \Sigma_{t-1} G_{ki} \tag{3.5}$$

where Σ_t denotes the conditional covariance matrix at time t, C_0 denotes an upper triangular matrix, D_{ki} and G_{ki} are parameter matrices, and ε_{t-1} is a vector of lagged disturbances originating from the subsample's corresponding VAR processes. The BEKK representation satisfies according to Hafner and Herwartz (2001, 2006) two issues evolving in multivariate GARCH-modelling: On the one hand one obtains a restricted multivariate GARCH model which generates always a positive definite covaricance matrix ε_{t-1} regardless the parameter estimates for D_{ki} and G_{ki}, respectively. On the other hand the BEKK representation allows for direct dependency of the conditional variance of one variable on past disturbances as well as past variances of the other variables within the system.

Comparing both subsamples volatility processes though, requires further assumptions. Since the parameter matrices of equation (3) exhibit super consistency, the disturbance vectors of both subsamples will be restricted such that $\widehat{\Sigma}_1 = \widehat{\varepsilon}_t^{*\prime}\widehat{\varepsilon}_t^{*}$ and $\widehat{\Sigma}_2 = \widehat{\varepsilon}_t^{*\prime}\widehat{\varepsilon}_t^{*}$ with

$$\widehat{\varepsilon}_t^{*} = Y_t - \widehat{\upsilon}_0 - \widehat{A}_1 Y_{t-1} - ... - \widehat{A}_L Y_{t-L} \tag{3.6}$$

for $t = 1, ..., T_1$ and $t = T_1 + 1, ..., T_2$.

The matrix $\widehat{\Sigma}_1$denotes the estimated covariance matrix of the first subsample running from $t = 1, ..., T_1$, whereas $\widehat{\Sigma}_2$ denotes the estimated covariance matrix concerning the second subsample running from $t = T_1 + 1, ..., T_2$. Consequently, the BEKK model estimates will be based on the restricted VAR model residuals of equation (6), as it is assumed that the stock markets' data generating processes with respect to the first order moment is constant over time. While the covariance matrix Σ_0 captures the overall volatility and covariance, $\Sigma_1$and Σ_2 use only the information of the corresponding subsamples.

In order to analyze volatility spillovers from one stock market to the other, volatility impulse response functions (VIRF) are employed as introduced by Hafner and Herwartz (2001, 2006). The volatility impulse response methodology accounts for shocks in volatility which is generated by the underlying data generating processes. Unlike the traditional impulse response analysis, the shock does not occur in the VAR error term ε_t^*, but in the *iid* error term ξ_t instead:

$$vech(\Sigma_t) = c + \sum_{i=1}^{p} A_i vech(\varepsilon_{t-1}^{*}\varepsilon_{t-1}^{*\prime}) + \sum_{i=1}^{q} B_i vech(\Sigma_{0,t-i}) \tag{3.7}$$

where $\varepsilon_t^* = \Sigma_t^{1/2}\xi_t$, and consequently for $q = p = 1$

$$V_{t=1}(\xi_0) = c + A_1 vech(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2}) + B_1 vech(\Sigma_0) \tag{3.8}$$

for $t = 1$ and

$$V_{t=2}(\xi_0) = c + (A_1 + B_1)(c + A_1 vech(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2}) + B_1 vech(\Sigma_0)) \tag{3.9}$$

for $t = 2$ and

$$V_{t\geq 2}(\xi_0) = c + (A_1 + B_1)V_{t-1}(\xi_0) \tag{3.10}$$

in general for $t \geq 2$. Determining the square root of the covariance matrix is necessary to calculate the volatility impacts concerning the term $vech(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2})$. Hafner and Herwartz (2006) suggest using the Jordan decomposition. The Jordan decomposition is for every matrix which is of full rank unique. Let λ_{ti} be the eigenvalues of the covariance matrix Σ_t with $\Sigma_t \in M_{N,N}$ and $i = \{1, ..., N\}$. Then, for each eigenvalue there is one corresponding eigenvector existing denoted by γ_{ti} with $\gamma_{ti} \in M_{N,1}$. As Σ_t is symmetric, $\Sigma_t^{1/2}$ is symmetric as well and due to the Jordon decomposition defined as

$$\Sigma_t^{1/2} = Z_t \Lambda_t^{1/2} Z_t' \tag{3.11}$$

where $Z_t \in \{\gamma_{t1}, ..., \gamma_{tN}\}$ and $\Lambda_t^{1/2} = diag(\lambda_{t1}, ..., \lambda_{tN})$ such that $\Sigma_t^{1/2}\Sigma_t^{1/2} = \Sigma_t$. In contrast to traditional impulse response analysis, by using the Jordan decomposition, it can be avoided to impose any zero restrictions. Moreover, shocks can be considered as they occur independently from each other since the innovation vector ξ_t is assumed to be multivariate normally distributed with $\xi_t \sim (0, I_N)$. The $\Sigma_t^{1/2}$ matrix links the multivariate normally distributed innovation vector with the disturbance vector ε_t^* and thus, generates the time varying correlations.

The VIRF methodology as introduced by Hafner and Herwartz (2006) allows considering, respectively, tracing volatility spillover effects of different shocks scenarios. Thereby, the shock vector is set equal to the initial shock scenario under consideration and the dynamic patterns of this shock can be traced over time. For instance, setting the vector $\xi_t' = (1, 0)$, the volatility spillover effects stemming from other stock market can be traced over time. However, in order to capture all possible shock scenarios, the volatility density has to be calculated such that $\xi_t' = (x, 0)$ with $x \in (-\infty, \infty)$, in principle. In contrast to the VIRF methodology, estimating the density functions allows to determine the overall volatility impact at all. Thereby, all possible shocks are weighted with the corresponding probability. Since Grobys (2009) points out that the corresponding conditional probability distribution of each shock occurring in the vector ξ_t for $\xi_t' = (x, 0)$ is chi-square distributed with one degree on freedom, the volatility density $\varphi(t, x)$ at time t given the initial shock $\xi_t' = (x, 0)$ with $x \in (-\infty, \infty)$ can be calculated by

$$\varphi_{t=1}(t, \xi_0) = c + \int_{x=-\infty}^{\infty} A_1 vech(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2}) f(x) + B_1 vech(\Sigma_0) \tag{3.12}$$

for $t = 1$,

$$\varphi_{t=2}(t, \xi_0) = c + \int_{x=-\infty}^{\infty} (A_1 + B_1)(c + A_1 vech(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2}) f(x) + B_1 vech(\Sigma_0)) \tag{3.13}$$

for $t = 2$,

$$\varphi_{t\geq 2}(t, \xi_0) = c + \int_{x=-\infty}^{\infty} (A_1 + B_1)\varphi_{t-1}(t, \xi_0) \tag{3.14}$$

for $t > 2$, where the function $f(x)$ weights each shock in the term $vech(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2})$ with its corresponding probability distribution being chi-square distributed with one degree of freedom. In other words, the volatility density as defined here is summing up all possible shocks which may occur at the stock market while taking into account the corresponding probability of each shock. As ξ_0 is assumed to be multivariate normally distributed, the conditional distribution (i.e. under the condition that the shock in the stock market under consideration is set equal to zero) will be normally distributed. In the term $vech(\Sigma^{1/2}\xi_0\xi_0'\Sigma^{1/2})$, however, the latter random variable is squared and as a consequence, chi-square distributed with one degree of freedom.

4. Limitations of the Dataset

Stock market data availability is limited especially concerning long-run horizons. The data set here contains high frequented stock market data which is available only from 26.11.1990-28.05.2010 with respect to the stock markets being analyzed. Data of the European stock indices DAX 30[2], FTSE 100[3] and CAC 40[4] are downloaded from yahoo.com being in line with Alexander and Dimitriu (2005) who also employ data from this data source which is available for free. However, stock market data for the index OMX 30[5] can be downloaded on the index

[2]The DAX 30 is the German's leading stock index containing the largest 30 companies in Germany.
[3]The FTSE 100 is the British's leading stock index containing the largest 100 companies in Great Britain.
[4]The CAC 40 is the French's leading stock index containing the largest 40 companies in France.
[5]The OMX 30 is the Swedish's leading stock index containing the largest 30 companies in Sweden.

provider's homepage nasdaqomxnordic.com. Index notations in different countries may deviate from each other due to red-letter days for instance. The whole sample accounting for 4792 daily observations is consequently adjusted such that for each accounted trading day every index exhibits a price notation. Moreover, the whole sample is divided such that sample 1 contains data from 26.11.1990-05.10.2000, whereas sample 2 contains data from 06.10.2000-28.05.2010. Thus, both samples contain 2396 observations corresponding to 10 years.

5. Results

The correlation-matrix (see exhibit 1) shows that the correlation between all stock indices has increased over time. The lowest increase in correlation is between the French and the German stock index being 29% and the largest relative change in correlation is between the British and the Swedish stock index which is 44%. Testing the whole sample pair wise for cointegration (see exhibit 2) shows that the German stock index has a cointegration relationship with the British and Swedish stock index. These results hold even if the trace-test statistic accounts for a trend term. The German and British stock markets exhibit for both samples a cointegration relationship as long as the trace test statistic accounts for a constant and trend term (i.e. p-value=0.03 and p-value=0.02). However, the German and the Swedish stock markets have a cointegration relationship for the second sample only, irrespective if the trace test statistic does account for a trend term or not (i.e p-value=0.00 for both samples). Considering the overall sample though, a cointegration relationship can be ascertained between the German and the Swedish stock market as well as the German and the British stock market, irrespective if the trace test statistic does account for a trend term or not. Against the background of the 20.05.2010, in the following, volatility spillover effects will be analyzed that arise at the German stock market and cross over other European stock markets. As the French stock market does not show a cointegration relationship with the German stock market, volatility spillovers to the Swedish and British stock markets are investigated, only. As the Schwarz Criterion suggests a lag-order of L=2, the estimated VAR models of the overall sample are as follows (t-values in parenthesis):

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \underset{(4.72)}{0.04} \\ \underset{(3.06)}{0.02} \end{bmatrix} + \begin{bmatrix} \underset{(46.31)}{0.94} & \underset{(2.86)}{0.06} \\ \underset{(4.02)}{0.09} & \underset{(47.23)}{0.96} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} \underset{(2.42)}{0.05} & \underset{(-2.44)}{-0.05} \\ \underset{(-4.29)}{-0.09} & \underset{(2.25)}{0.05} \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{2t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{5.1}$$

where $Y_t = (\log(DAX_t), \log(OMX_t))'$ and

$$\begin{bmatrix} y_{1t} \\ y_{2t} \end{bmatrix} = \begin{bmatrix} \underset{(-2.30)}{-0.03} \\ \underset{(0.87)}{-0.01} \end{bmatrix} + \begin{bmatrix} \underset{(44.43)}{0.96} & \underset{(1.27)}{0.04} \\ \underset{(3.37)}{0.06} & \underset{(42.67)}{0.92} \end{bmatrix} \begin{bmatrix} y_{1t-1} \\ y_{2t-1} \end{bmatrix} + \begin{bmatrix} \underset{(1.43)}{0.03} & \underset{(-0.95)}{-0.03} \\ \underset{(-3.42)}{-0.06} & \underset{(3.51)}{0.08} \end{bmatrix} \begin{bmatrix} y_{1t-2} \\ y_{2t-2} \end{bmatrix} + \begin{bmatrix} \varepsilon_{1t} \\ \varepsilon_{2t} \end{bmatrix} \tag{5.2}$$

where $Y_t = (\log(DAX_t), \log(FTSE_t))'$. Since cointegration between DAX and OMX, respectively, DAX and FTSE holds (see exhibit 2), the parameter matrices A_1 and A_2 of both equations are super-consistent and therefore used to estimate $\widehat{\Sigma}_1 = \widehat{\varepsilon}_t^{*\prime}\widehat{\varepsilon}_t^*$ and $\widehat{\Sigma}_2 = \widehat{\varepsilon}_t^{*\prime}\widehat{\varepsilon}_t^*$ for both models. The covariance matrices $\widehat{\Sigma}_1$ (i.e. covariance matrix of sample 1) and $\widehat{\Sigma}_2$ (i.e. covariance matrix of sample 2) are allowed to vary over time. The BEKK-model parameter matrices (see appendix) of both samples are employed to estimate the volatility impulse response functions (VIRF) for each sample. Thereby, the volatility state Σ_0 of equations (8)-(10) is for both samples chosen with respect to the unconditional covariance matrix of the whole sample, as given by equation (3), respectively equations (15) and (16). Thus, the volatility shocks being integrated have the same origin, respectively, the same state of the system. Integrating all possible shocks while accounting for the probability being chi-square distributed with one degree of freedom leads to the volatility impulse response density functions as shown in figures 1 and 2, respectively. Both stock markets show an increase in volatility spillover effects. Taking into account 75 trading days, the volatility spillover effects concerning the Swedish stock market increased 73.87% when comparing the time windows 26.11.1990-05.10.2000 and 05.10.2000-28.05.2010, while the spillover effects with respect to the British stock market increased only 15.52% during the same periods.

Figure 1. Volatility impulse response density function DAX-OMX

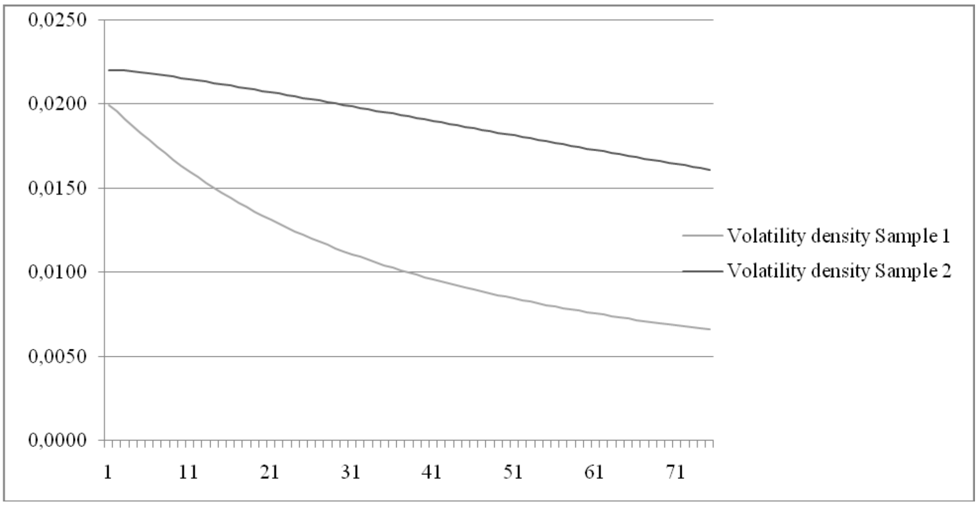

Figure 2. Volatility impulse response density function DAX-FTSE

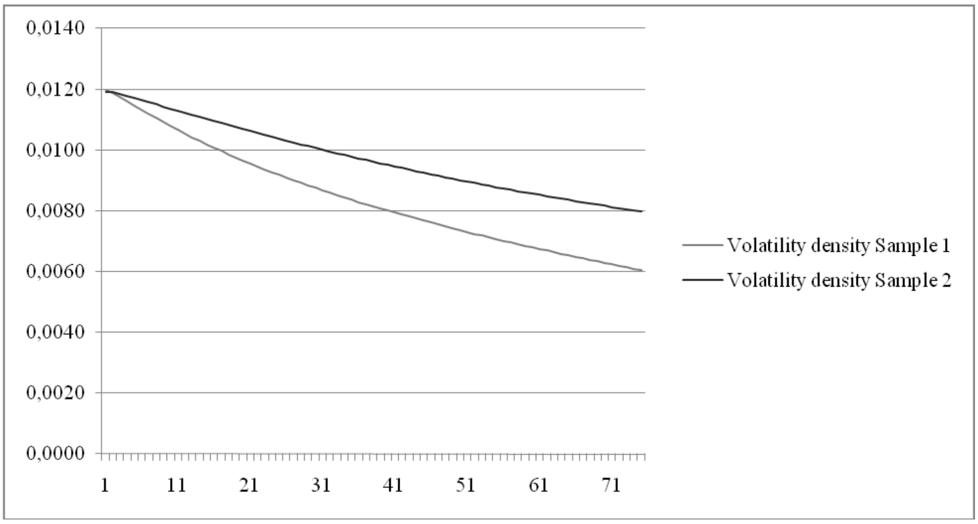

Table I. Correlation-matrix of European stock indices

Correlation Matrix		DAX	FTSE	CAC	OMX
	Sample: 26.11.1990-28.05.2010	1.00	0.74	0.80	0.69
		0.74	1.00	0.82	0.69
		0.80	0.82	1.00	0.71
		0.69	0.69	0.71	1.00
	Sample 1: 26.11.1990-05.10.2000	1.00	0.60	0.68	0.58
		0.60	1.00	0.68	0.55
		0.68	0.68	1.00	0.58
		0.58	0.55	0.58	1.00
	Sample 2: 06.10.2000-28.05.2010	1.00	0.81	0.88	0.77
		0.81	1.00	0.91	0.79
		0.88	0.91	1.00	0.82
		0.77	0.79	0.82	1.00
	Relative change of correlation	-	35%	29%	33%
		35%	-	34%	44%
		29%	34%	-	41%
		33%	44%	41%	-

6. Discussion

The results of Gklezakou and Mylonakis (2010) that correlations between different economies' stock markets have increased empirically over time can be supported. However, Gklezakou and Mylonakis (2010) do not focus on European stock markets only. Furthermore, they employ logarithmic daily closing prices from 01.01.2000 to 20.02.2009 including 2385 daily observations, whereas the study here accounts for a time horizon of 20 years. Furthermore, Gklezakou's and Mylonakis' (2010) findings verify previous studies suggesting that the DAX seriously affect other indices independent of the prevailing bear or bull market conditions. However, it could be shown here that these effects are even related to the second order moment, respectively, the stock-markets' volatilities.

Furthermore, Pascual's (2003) findings that cointegration tests do not show evidence of changes in the degree of financial integration for the British and German stock market can only be supported if the trace-test statistic includes a trend term (i.e. p-value=0.03 in sample 1 and p-value=0.02 in sample 2). Considering sample 1, the trace-test statistic suggest that the Swedish stock market and the German stock market are not cointegrated, irrespective if it is accounted for a trend term or not (i.e. p-value=0.46 and p-value=0.07). The second sample, though suggests the opposite namely that the German and Swedish stock markets are cointegrated. Consequently, the integration of other European stock markets (i.e. the Swedish stock market) can be stated, too. Exhibit 2 shows also that the Swedish and the French stock markets show a cointegration relationship concerning the second subsample as long as the test statistic accounts for a trend term (i.e. p-value=0.05).

Panopoulou and Pantelidis (2005) investigate the international information transmission between the U.S. and the rest of the G-7 countries and use daily stock market return data covering 20 years (31.12.1985-08.10.2004) including 4896. A similar approach is taken into account here while accounting for daily observations from 26.11.1990-28.05.2010 including 4792 observations after adjusting the data set. However, Panopoulou and Pantelidis (2005) model the first order moment by employing log-returns while the Vector-Auto-Regession (VAR) model being employed in this study takes into account the log-prices involving lagged log-prices of lag-order L=2. This approach makes sense in this context because OLS-parameter estimates of cointegrated time series exhibit super consistency and thus, are more robust.

Furthermore, the parameter matrices being used to estimate both samples covariance matrices (see equation 6) are restricted with respect to the parameter estimates of the whole sample in order to make the models comparable, whereas Panopoulou and Pantelidis (2005) take initial values for the estimation of the BEKK models by setting no further restrictions. Moreover, they divide the whole sample in two subsample of equal length where the first sample ends on the 31.12.1994. The same approach is taken into account in this study at hand where the first sample ends on 05.10.2000. While Panopoulou and Pantelidis (2005) find out that volatility spillover effects stemming from the US-stock markets have increased over time, the study here shows that volatility spillovers from the German to the Swedish and British stock markets have increased over time, too. Consequently, both sample-split analyses reveal that the linkages between stock markets have changed substantially.

Table II. Testing European stock indices for Cointegration

Matrix of Trace-test p-values (constant term)		**DAX**	**FTSE**	**CAC**	**OMX**
	Sample: 26.11.1990-28.05.2010	0.00	0.00	0.83	0.00
		0.00	0.00	0.10	0.07
		0.83	0.10	0.00	0.10
		0.00	0.07	0.10	0.00
	Sample 1: 26.11.1990-05.10.2000	0.00	0.04	0.70	0.70
		0.04	0.00	0.04	0.13
		0.70	0.04	0.00	0.38
		0.70	0.13	0.38	0.00
	Sample 2: 06.10.2000-28.05.2010	0.00	0.17	0.70	0.00
		0.17	0.00	0.55	0.05
		0.70	0.55	0.00	0.34
		0.00	0.05	0.34	0.00
Matrix of Trace-test p-values (constant and trend term)		**DAX**	**FTSE**	**CAC**	**OMX**
	Sample: 26.11.1990-28.05.2010	0.00	0.00	0.99	0.01
		0.00	0.00	0.30	0.13
		0.99	0.30	0.00	0.28
		0.01	0.13	0.28	0.00
	Sample 1: 26.11.1990-05.10.2000	0.00	0.03	0.92	0.46
		0.03	0.00	0.24	0.35
		0.92	0.24	0.00	0.89
		0.46	0.35	0.89	0.00
	Sample 2: 06.10.2000-28.05.2010	0.00	0.02	0.31	0.00
		0.02	0.00	0.14	0.00
		0.31	0.14	0.00	0.05
		0.00	0.00	0.05	0.00

Volatility impulse response functions (VIRF) as introduced by Hafner and Herwartz (2001, 2006) and applied by Panopoulou and Pantelidis (2005), for instance, trace a specific shock in the dynamic system of financial time series data. Thereby, the shock vector ξ_0 is set to the initial shock and then the dynamic features are figured out. This concept though lacks in the involved probability of each shock. Of course, given that a shock occurred, VIRFs may provide a forecast of how the persistency concerning the following periods might be. However, it is not accounted for how high the probability of such shocks actually may be. The concept of volatility impulse response density functions (VIRDF), as suggested here does not provide a forecast of how the dynamics of a specific isolated shock will be, but it displays rather the overall impact of volatility spillovers from one specific market to the other. However, the methodology as being introduced here assumes that the underlying stochastic process is stable and not changing in both subsamples (see equation 6). Even though the whole sample shows a cointegration relationship between the stock markets being considered, the previous section reveals that this may not be true for all stock markets. For instance, only the DAX and the FTSE show a stable cointegration relationship for both subsamples as long as a trend term is included (see exhibit 2), whereas the DAX and the OMX show a cointegration relationship for the second subsample only.

7. Concluding Remarks

The correlations between European stock markets have increased over time. Cointegration though can be considered as powerful statistical tool that, in a sense, generalizes the concept of correlation to non-stationary time series. Three of four analyzed European stock markets are cointegrated which means that they follow the same stochastic trends. Consequently, the home-bias problem cannot be solved by investing in other European countries any longer. Furthermore, examining the second order moment shows that volatility spillover effects increased over time, too. The Swedish stock market shows that volatility spillover effects stemming from the German stock market are during the last decade 73.87% larger compared to the decade before.

Figuring out changes concerning the second order moment requires different assumptions. First, the first order moment being estimated must be valid and stable over time. In this analysis, the stock markets log-prices are employed in order to estimate VAR-models. Employing log-prices instead of the log-returns may be justified by a cointegration relationship. Consequently, VAR-model parameter matrices will exhibit estimated parameter matrices being super consistent. Restricting both samples to those parameter matrices makes sense because it is assumed that the stochastic process generating the first order moment is not changing over time. Second, the volatility state being chosen to estimate the VIRF is consistent with the unconditional covariance matrix of the overall sample. Consequently, the BEKK-model parameter matrices involve substantial information being related to the restricted disturbances (i.e. stemming from the restricted VAR-models of each sample).

In contrast to ordinary VIRF, integrating all possible volatility shock scenarios while accounting for the corresponding probability makes it possible to determine the overall volatility impact from one stock market to the other. This concept can also be applied to discover volatility impacts in other kind of financial markets. The knowledge of volatility impacts between stock markets may be an important issue in financial management that aims to minimize risks. Increasing volatility spillover impacts makes it more difficult to diversify away portfolio risk, for instance. A route for further investigation may be the extension of this isolated bivariate analysis to a higher order one, allowing for interactions among three or more stock-markets.

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Appendix

A 1) BEKK-model parameter estimates DAX-OMX sample 1

D	G
$\begin{bmatrix} \underset{(10.11)}{2.24e-01} & \underset{(1.27)}{2.88e-02} \\ \underset{(0.99)}{2.08e-02} & \underset{(11.91)}{2.61e-01} \end{bmatrix}$	$\begin{bmatrix} \underset{(94.68)}{9.50e-01} & \underset{(-2.55)}{-2.86e-02} \\ \underset{(-0.39)}{-3.51e-03} & \underset{(100.41)}{9.53e-01} \end{bmatrix}$
C_0	Modulus of Eigenvalues
$\begin{bmatrix} \underset{(11.47)}{1.23e-03} & \underset{(9.13)}{1.06e-03} \\ \underset{(0.00)}{0.00} & \underset{(5.66)}{6.43e-04} \end{bmatrix}$	$9.61718e-01$ $9.61718e-01$ $9.61718e-01$ $9.62660e-01$

Note: p-values in parenthesis.

A 2) BEKK-model parameter estimates DAX-OMX sample 2

D	G
$\begin{bmatrix} \underset{(13.93)}{3.43e-01} & \underset{(-3.03)}{-6.98e-02} \\ \underset{(-1.12)}{-2.59e-02} & \underset{(14.53)}{3.17e-01} \end{bmatrix}$	$\begin{bmatrix} \underset{(88.65)}{9.25e-01} & \underset{(2.82)}{2.95e-02} \\ \underset{(0.97)}{7.25e-03} & \underset{(112.90)}{9.38e-01} \end{bmatrix}$
C_0	Modulus of Eigenvalues
$\begin{bmatrix} \underset{(16.66)}{1.17e-03} & \underset{(5.47)}{5.47e-04} \\ \underset{(0.00)}{0.00} & \underset{(6.32)}{4.62e-04} \end{bmatrix}$	$9.76915e-01$ $9.76915e-01$ $9.79799e-01$ $9.74877e-01$

Note: p-values in parenthesis.

A 3) BEKK-model parameter estimates DAX-FTSE sample 1

D	G
$\begin{bmatrix} \underset{(9.78)}{2.13e-01} & \underset{(3.34)}{5.02e-02} \\ \underset{(0.53)}{1.93e-02} & \underset{(9.02)}{2.15e-01} \end{bmatrix}$	$\begin{bmatrix} \underset{(86.55)}{9.40e-01} & \underset{(-4.13)}{-3.50e-02} \\ \underset{(1.70)}{2.54e-02} & \underset{(98.25)}{9.83e-01} \end{bmatrix}$
C_0	Modulus of Eigenvalues
$\begin{bmatrix} \underset{(11.62)}{1.20e-03} & \underset{(5.71)}{4.99e-04} \\ \underset{(0.00)}{0.00} & \underset{(1.91)}{2.85e-04} \end{bmatrix}$	$9.68356e-01$ $9.68356e-01$ $9.73438e-01$ $9.69022e-01$

Note: p-values in parenthesis.

A 4) BEKK-model parameter estimates DAX-FTSE sample 2

D	G
$\begin{bmatrix} \underset{(15.65)}{3.90e-01} & \underset{(3.57)}{6.88e-02} \\ \underset{(-5.03)}{-1.50e-01} & \underset{(10.10)}{2.52e-01} \end{bmatrix}$	$\begin{bmatrix} \underset{(100.03)}{9.02e-01} & \underset{(-4.97)}{-3.78e-02} \\ \underset{(7.42)}{7.46e-02} & \underset{(107.48)}{9.75e-01} \end{bmatrix}$
C_0	Modulus of Eigenvalues
$\begin{bmatrix} \underset{(8.94)}{7.46e-04} & \underset{(7.82)}{5.98e-04} \\ \underset{(0.00)}{0.00} & \underset{(6.92)}{3.47e-04} \end{bmatrix}$	$9.92024e-01$ $9.77804e-01$ $9.77804e-01$ $9.91663e-01$

Note: p-values in parenthesis.