MARKOWITZ VERSUS REGIME SWITCHING: AN EMPIRICAL APPROACH

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ABSTRACT. This article discusses an adjusted regime switching model in the context of portfolio optimization and compares the attained portfolio weights and the performance to a classical mean-variance set-up as introduced by Markowitz (1952). The model postulates different asset price dynamics under different regimes, and jumps between regimes are driven by a Markov process. For examples, 'bear' and 'bull' markets could be such regimes. Given a particular regime, portfolio weights are set based on the conditional means and variancecovariance structure of the asset dynamics. The model is evaluated in an out-of-sample period of the last three years with a moving window and a forecast of only one period. It is found that with the adjusted regime switching portfolio selection algorithm as applied here, the performance of the optimal portfolio is highly improved even where portfolio weights are constrained to realistic values.

1. INTRODUCTION

Classical portfolio theory as introduced by Markowitz has to be reassessed when considering the returns of major asset classes over the last two decades. Markowitz assumes that asset returns follow a stationary normal distribution with constant correlation matrix. Hong and Campbell show that this assumption is not any more supported by empirical data. In fact the correlation in asset class returns during a 'bear' market period will be much higher than in a 'bull' market, such that portfolio diversification solely based on Markowitz will no longer be satisfactory. Moreover the assumption that asset class returns follow a normal distribution is not supported by observed market data. Many studies, such as Levy (1969) and Samuelson (1970), found that asset class returns are skewed and fat tailed and, therefore, not normally distributed.

Mandelbrot showed that extreme negative returns occur too often to support the normal distribution assumption. One way to overcome these issues for the purpose of portfolio optimization would be to take into account higher moments of the portfolio return. Alternatively the data series can be divided into a number of regimes with different dynamics in each. Since Glawischnig find that including the third and fourth moment does not majorly affect the optimal portfolio weights, the focus is on the approach of regime switching.

Quandt was first to introduce a regime switching framework with a maximum likelihood estimation procedure. In the absence of capable computers, the points of switching were evaluated by hypothesis tests. Subsequently Goldfeld further developed the model of Quandt to allow for more switches between states. They used for the first time a Markov chain framework to model regime switches.

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Hamilton (1989) further developed the Markov chain framework for regime switching and introduced an iterative approach to calculating the persistence matrix, the regimes and the regime changes via a maximum likelihood function.

In the last years regime switching processes have received increased interest. This is due to a general view to incorporate the non-normality of asset returns into models, fuelled by the financial crises of the last decade. Recent empirical studies, such as Ang (2002a), Ang (2002b) and Grobys, pointed out the advantages of dividing the whole time series into two or more regimes. The time series will often be divided into a 'bear' (higher volatility, higher correlation and lower conditional means) and a 'bull' market (lower volatility, lower correlation and a higher conditional means), as discussed in Ang (2002a). We acknowledge that Markowitz portfolio diversification has shortcomings particularly in 'bear' markets, which has been empirically documented by Karolyi.

Dividing the time series into two or more regimes allows to still model conditional asset returns as normally distributed while achieving a significantly better fit of the time series to market data.

Frauendorfer (2007) and Chen present regime-dependent regression factor models, and they point out suitable explanatory variables to obtain the regimes via regression analysis.

Resent studies such as Yin, Wu and Coster developed a closed form of the mean-variance portfolio selection with regime switching. In this article the classical Markowitz portfolio diversification and a model taking a regime switching approach into account are compared. Up to this paper a comparison with empirical data has not been done.

This article is structured as follows. Section 2 discusses the major parts of the regime switching model of Hamilton (1989). In Section 3 the model of the regime switching process and the corresponding algorithm which is implemented will be discussed. Sections 4 and 5 describe the used data set and conduct various statistical tests. The empirical results of the out-of-sample analysis will be shown in Section 6, and Section 7 concludes the paper.

2. Model

Following Hamilton (2005) we suppose that a return time series until time $t = 1, 2, 3, ..., t_0$ can be described by a first-order autoregression,

$$r_t = c_1 + \phi r_{t-1} + \epsilon_t,$$

with $\epsilon_t \sim N(0, \sigma^2)$. At time t_0 a fundamental change occurs in the market such that from that time on the time series follows the dynamics

$$r_t = c_2 + \phi r_{t-1} + \epsilon_t.$$

For simplification the two equations can be composed to

$$r_t = c_{S_t} + \phi r_{t-1} + \epsilon_t,$$

with $\epsilon \sim N(0, \sigma_{S_t}^2)$ and $c_{S_t} \in \{c_1, c_2\}$.

The switching process from one regime to another will be modelled by a two-state Markov chain such that

$$P(S_t = j | S_{t-1} = i, S_{t-2} = k, \dots, y_{t-1}, y_{t-2}, \dots) = P(S_t = j | S_{t-1} = i) = p_{ij}.$$
(2.1)

Note that the regime at time t only depends on the regime at time t-1 and not on the whole history of the time series. Suppose that only y_t can be observed, while one can make an inference about S_t . This inference can be formulated based on (2.1) as

$$\xi_{j,t} = P(s_t = j | \Omega_t; \theta), \tag{2.2}$$

with j = 1, 2 and $\xi_{1,t} + \xi_{2,t} = 1$. Let $\Omega_t = \{y_t, y_{t-1}, \dots, y_1, y_0\}$ be the set of observations and let $\theta = (\sigma, \phi, c_1, c_2, p_{11}, p_{22})$ be the parameter vector of the model. The inference will be calculated iteratively as

$$\xi_{j,t-1} = P(s_{t-1} = j | \Omega_{t-1}; \theta), \tag{2.3}$$

for j = 1, 2. The main idea of the algorithm is that the conditional densities of the two regimes follow a normal distribution, i.e.

$$\eta_{jt} = f(y_t|s_t = j, \Omega_{t-1}; \theta) = \frac{1}{\sqrt{2\pi\sigma}} \exp\left(-\frac{(y_t - c_j - \phi y_{t-1})^2}{2\sigma^2}\right),$$
(2.4)

for j = 1, 2. Using (2.3), the conditional density of the t-th observation is calculated by

$$f(y_t|\Omega_{t-1};\theta) = \sum_{i=1}^2 \sum_{j=1}^2 p_{ij}\xi_{i,t-1}\eta_{jt}$$
(2.5)

to yield

$$\xi_{j,t} = \frac{\sum_{i=1}^{2} p_{ij} \xi_{i,t-1} \eta_{jt}}{f(y_t | \Omega_{t-1}; \theta)} P(s_t = j | \Omega_t; \theta).$$
(2.6)

Subsequent to the calculation of all conditional densities for all observations, θ can be calculated by maximizing the conditional log likelihood function

$$\log f(y_1, y_2, \dots, y_T | y_0; \theta) = \sum_{t=1}^T \log f(y_t | \Omega_{t-1}; \theta).$$

Now the only additional thing required is the starting value of $\xi_{j,t}$. We apply the matlab packages provided by Perlin and, therefore, follow his method of using $\xi_{j,0} = \frac{1}{2}$ as starting value.

The optimization problem is then formulated by the Markowitz two-moment function

$$\max_{w} w' \mu_{s_t} - \lambda w' \Sigma_{s_t} w \tag{2.7}$$

subject to

$$\mathbf{1}'w = 1, \mathbf{1}' = [1, 1, \dots, 1],$$

where μ_{s_t} denotes the mean vector of the asset returns and Σ_{s_t} the variance-covariance matrix dependent on state S_t . Note that the optimal portfolio is chosen by penalising the expected return by a multiple of the portfolio variance, and λ will be the risk-aversion coefficient of the investor.

3. Algorithm

When applying the algorithm as provided by Perlin, we failed to produce an estimate of the parameters via maximizing the likelihood function with five asset classes. This appears to be due to the large dimension of θ , as with five equations and three explanatory variables over 80 parameters have to be estimated, such that the gradient descent optimization method, as implemented in the fmincon function of matlab, fails. For that reason and following Chen the choice of asset classes is restriced to stocks and bonds for the estimation of θ .

In the following a multivariate framework is chosen, with stock and bond returns as dependent variables. The mean return, the standard deviation and the correlation between stocks and bonds, over the most recent three months, are chosen as independent variables. The focus is, that regimes are driven mostly by stock and bond returns and their correlation and therefore, after the estimation of the regimes the model can be easily extended to five asset classes like we do in Step 4 of the algorithm. Let

$$rStock_{t} = \phi_{1,1}rStock_{t-1} + \phi_{1,2}x_{1,t} + \phi_{1,3}x_{2,t} + \phi_{1,4}x_{3,t} + \epsilon_{t} \text{ and}$$
(3.1)
$$rBond_{t} = \phi_{2,1}rBond_{t-1} + \phi_{2,2}x_{1,t} + \phi_{2,3}x_{2,t} + \phi_{2,4}x_{3,t} + \epsilon_{t},$$

where

$$\begin{aligned} x_{1,t} &= \mu_{rStock_3, rBond_3} = \frac{1}{6} \sum_{i=1}^{3} (rStock_{t-i} + rBond_{t-i}), \\ x_{2,t} &= \sigma_{rStock_3, rBond_3} \\ &= \sqrt{\frac{1}{5} \sum_{i=1}^{3} (rStock_{t-i} - \overline{rStock})^2 + \sum_{i=1}^{3} (rBond_{t-i} - \overline{rBond})^2} \text{ and} \\ x_{3,t} &= \rho_{rStock_3, rBond_3} \\ &= \frac{1}{6\sigma_{rStock}\sigma_{rBond}} \sum_{i=1}^{3} (rStock_{t-i} - \overline{rStock})(rBond_{t-i} - \overline{rBond}). \end{aligned}$$

Hereby, $rStock_3$ and $rBond_3$ denote the stock and bond returns, respectively, of the most recent three months (t-3, t-2, t-1), and $\epsilon_t \sim N(0, \sigma_{S_*}^2)$.

To apply the two-moment optimization function of Markowitz, estimators of the means and the variance-covariance matrices of the asset classes are required. For the classical optimization case, the arithmetic mean x is taken as estimator of the mean, and the covariance for our variance-covariance matrix. For the regime switching framework the mean and the variancecovariance matrix conditional on the prevailing state is estimated. Hence, we can compute

$$\mu_{s_t} = \frac{1}{n_{s_t}} \sum_{i=1}^{n_{s_t}} x_i, \tag{3.2}$$

$$\Sigma_{s_t} = (\text{Cov}(x_i, x_j))_{i,j=1,...,n_{s_t}}.$$
 (3.3)

The optimal portfolio weights at time t + 1 are then determined as

$$upw(t) = \xi_{1t}(P(1,1)w_{1t} + (1 - P(1,1))w_{2t}) + (1 - \xi_{1t})(P(2,2)w_{2t} + (1 - P(2,2))w_{1t}),$$
(3.4)

with w_{it} being the optimal portfolio weights conditional on regime *i*.

The following algorithm calculates the optimal Markowitz portfolios conditional upon the regimes:

- Step 1: Calculate θ by using the model as in (3.1) up to time t.
- Step 2: Take each ξ_{ik} for $k = 1, \dots, t$ calculated by (2.6) and compare it with 0.5.
- Step 3: This divides the whole in-sample period into two parts, one per each regime.
- Step 4: Expand the two asset classes used for the estimation of the regimes up to five, adding the returns of hedge funds, real estate and commodities to the dataset.
- Step 5: Estimate μ_i and Σ_i and calculate the optimal portfolio weights dependent on the regime.
- Step 6: Compute the unconditioned portfolio weights *upw* as per (3.4).
- Step 7: Calculate the expected portfolio mean return by the product of the asset returns at time t + 1, times the unconditioned portfolio weights at time t.
- Step 8: Set t = t + 1.

4. Data

For the empirical investigation, the data set contains a time series with monthly data from January 1999 until June 2011. The investment universe consists of the major five asset classes. The following indices are considered to describe the returns of the different asset classes: MSCI World (stocks), Barclays Global Aggregate (bonds), C.S/Tremont Hedge Fund net asset value (hedge funds), FTSE EPRA/NAREIT Developed \$ total return (real estate) and the TR Equal

Weight CCI (commodities), with 146 price observations each. All data was obtained from Thomson Reuters Datastream. For each index, the rate of return is calculated as follows,

$$r_{it} = \ln\left(\frac{p_{it}}{p_{it-1}}\right),\,$$

where p_{it} states the price of index *i* at time *t*.

5. Statistical Tests

Shapiro-Wilk-Test. We start with a hypothesis test whether the returns follow a normal distribution or not. For that reason and because we have in Regime 2 a small data sample n < 50 we conduct the Shapiro-Wilk test, which is a robust test also for small data samples.

Table 1 shows that only under Regime 2, H_0 will not be rejected for both bonds and stocks at $\alpha = 5\%$. Hence we assume that our data sample is not normally distributed. At the $\alpha = 1\%$ level, we would choose to reject only the normality assumption for Hedge Funds. So we get for at least Regime 2 a much better fit under the normality assumption.

Data	Stocks	Bonds	Hedge Funds	Commodities	Real Estate
W	0.9576	0.9922	0.9265	0.9403	0.8591
p-value	0.0001564	0.5962	6.165 e-07	5.9e-06	1.265 e-10
Regime 1	Stocks	Bonds	Hedge Funds	Commodities	Real Estate
W	0.9576	0.9957	0.9387	0.9607	0.8339
p-value	0.001871	0.985	0.0001002	0.003205	1.462e-09
Regime 2	Stocks	Bonds	Hedge Funds	Commodities	Real Estate
W	0.9508	0.9641	0.8966	0.9253	0.9349
p-value	0.1027	0.2707	0.00237	0.01604	0.03186

TABLE 1. Values from the Shapiro-Wilk-Test

Pearson's Product Moment Correlation Test. With the Pearson correlation coefficient r the hypothesis whether the returns are correlated or not is tested.

In Table 2 it can be seen that the data subsets of Regime 1 and Regime 2 show both higher and lower correlations for asset class pairs, so that no clear correlation pattern is identified. Under general regime switching theory, as, for example, discussed by Ang (2002a), one would expect to find one regime with high correlations and the other one with low correlations across most asset class pairs. Furthermore five cases of significant p-values for the correlations in Regime 1 and four cases in Regime 2 in which it cannot be rejected H_0 . Hence, this theory could not be proved because we do not have a regime with significantly higher or significantly lower correlations compared to the other.

In Table 3 it can be seen that the standard deviation under Regime 2 is higher than the standard deviation across the whole dataset, and also higher than in Regime 1. Following Ang (2002a) this should be the 'bear' market regime. But like pointed out before it cannot be identified any significant difference in correlation between the asset classes in Regime 1 or 2. Hence the theory, that 'bear' markets coincide high volatility and high correlation between the asset classes could not be approved. One reason for this effect could be the construction of the model. The two regimes are evaluated by mean and volatility differences. In model (3.1) the correlation will also be taken into account, but do not effect the regimes as much as the mean and the volatility differences do.

Data	Stocks	Bonds	Hedge Funds	Commodities	Real Estates
Stocks	1.00	0.00 (0.99)	0.61(0.00)	0.47(0.00)	0.79(0.00)
Bonds	0.00	1.00	0.11 (0.41)	0.16 (0.18)	0.27(0.01)
Hedge Funds	0.61	0.11	1.00	0.57(0.00)	0.53(0.00)
Commodities	0.47	0.16	0.57	1.00	0.49(0.00)
Real Estates	0.79	0.27	0.53	0.49	1.00
Regime 1	Stocks	Bonds	Hedge Funds	Commodities	Real Estates
Stocks	1.00	0.06 (0.51)	0.55(0.00)	0.25 (0.05)	0.68(0.00)
Bonds	0.06	1.00	0.18 (0.24)	0.18 (0.24)	0.39(0.00)
Hedge Funds	0.55	0.18	1.00	0.34(0.00)	0.32(0.00)
Commodities	0.25	0.18	0.34	1.00	0.16 (0.24)
Real Estates	0.68	0.39	0.32	0.16	1.00
Regime 2	Stocks	Bonds	Hedge Funds	Commodities	Real Estates
Stocks	1.00	-0.01 (1.00)	0.68(0.00)	0.61(0.00)	0.85(0.00)
Bonds	-0.01	1.00	0.07 (1.00)	0.16 (1.00)	0.20 (0.95)
Hedge Funds	0.68	0.07	1.00	0.85(0.00)	0.73(0.00)
Commodities	0.61	0.16	0.85	1.00	0.72(0.00)
Real Estates	0.85	0.20	0.73	0.72	1.00

Data	Stocks	Bonds	Hedge Funds	Commodities	Real Estate
n	143	143	143	143	143
mean	0.0017	0.0049	0.0067	0.0101	0.0046
sd	0.0478	0.0214	0.0189	0.0601	0.0658
\min	-0.1764	-0.0528	-0.0678	-0.2931	-0.3263
\max	0.1249	0.0626	0.0818	0.1519	0.1831
skew	-0.7845	-0.1132	-0.3895	-1.1954	-1.5503
kurtosis	1.3740	0.3663	4.3443	4.0657	6.7261
Regime 1	Stocks	Bonds	Hedge Funds	Commodities	Real Estate
n	106	106	106	106	106
mean	0.0064	0.0033	0.0091	0.0126	0.0087
sd	0.0318	0.0192	0.0167	0.0470	0.0459
\min	-0.1299	-0.0476	-0.0474	-0.1106	-0.2359
\max	0.0774	0.0623	0.0818	0.0989	0.0707
skew	-0.8873	0.0202	0.6502	-0.4727	-2.1184
kurtosis	2.5450	0.2784	4.1776	-0.4943	7.9212
Regime 2	Stocks	Bonds	Hedge Funds	Commodities	Real Estate
n	37	37	37	37	37
mean	-0.0117	0.0096	-0.0001	0.0029	-0.0071
sd	0.0764	0.0264	0.0230	0.0880	0.1037
\min	-0.1764	-0.0528	-0.0678	-0.2931	-0.3263
\max	0.1249	0.0626	0.0398	0.1519	0.1831
skew	-0.1334	-0.4992	-1.1326	-1.0980	-0.7412
kurtosis	-1.0584	0.4359	2.4829	2.5330	2.0543

TABLE 3. Summary Statistics

6. Empirical Results

For the regime estimation based on the algorithm of Perlin the MSCI WORLD and BAR-CLAYS GLOBAL AGGREGATE indices are selected as regime dependent variables. The mean

and the variance-covariance matrix conditioned on the two regimes for the five asset classes as per equations (3.2) and (3.3) is estimated. Subsequently the quadratic program (2.7) with two different risk-aversion coefficients is maximized. For a less risk-averse investor $\lambda = 1$ is chosen and $\lambda = 5$ for a more risk-averse investor.

For $\lambda = 1$ one faces the problem that the optimal portfolio in the regime switching model includes short-selling of bonds at Jan-11 with a weight of about 800% and of stocks at Oct-10 of also approximately 600%. Note, however, that also the classical Markowitz portfolio would include short-selling weights of up to 500% based on the used dataset. This is illustrated in Figures 2 and 1. Short-selling weights of 500% or more would not be practicable in reality. In reality the model would be restricted to a practicable amount of short-selling, but with this restriction the optimal portfolio cannot be found any longer. In this paper we focus on the optimal solutions and therefor other practical optimal solutions with $\lambda = 5$ can be found.

Hence, the focus is on results for $\lambda = 5$, reflecting a more risk-averse investor, which yield more realistic results. As time horizon for the out-of-sample analysis it is choosen the period from July 2008 until June 2011. The in-sample period commences with April 1999. Forecasts are computed only for one month in advance as described in Section 3. Despite the issue around the temporarily large short-selling weights, Figure 4 depicts the performance of our regime-adjusted (RS) portfolio and the classical Markowitz portfolio for $\lambda = 5$. It can be seen that the RS-portfolio gains approximately 160% whereas the classical Markowitz portfolio loses about 20% over the considered time horizon.

Figure 3, where $\lambda = 1$, the RS portfolio increases in value to 260%, while the classical Markowitz portfolio even loses approximately 80%. Comparing the corresponding portfolio weights, Figures 1 and 5 describe a very smooth process whereas Figures 2 and 6 show a more scraggy process. Hence, in the absence of transaction costs the attained optimal portfolio would seem practicable. For further research it would be interesting to investigate the effect of transaction costs on the difference in performance of the RS and the classical Markowitz portfolio.



As expected the weights, both for $\lambda = 1$ and $\lambda = 5$, differ greatly for the examined portfolios. Also note that for $\lambda = 5$ the RS-portfolio does not require greater short-selling amounts than the calssical Markowitz portfolio. For both portfolios the maximum short-selling weight is about

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100% as can be seen in Figure 5 at Aug-08 for Real Estate and in Figure 6 at Sep-10 for Stocks. For a risk-averse investor the portfolio structure does not differ greatly as expected.











7. Conclusion

This paper shows in an out-of-sample analysis the differences in performance and portfolio weights of a classical Markowitz approach and a regime dependent portfolio optimization. The objective function for both methods is the mean-variance criterion which will be consistent with the maximization of a quadratic utility function (see Huang). The moving in- and out-of-sample analysis forecasts only one period to the future and, therefore, does not violate the Markov property. The model outperforms the classical Markowitz portfolio for both a risky and a risk averse investor, as applied in our analysis. A negative aspect regarding the performance will be the change in portfolio weights which can be seen between Figure 1 and 2 and Figure 5 and 6. At the classical mean-variance optimization it can be seen a very smooth run of the different asset weights, whereas at the regime dependent portfolio optimization the asset weights differ of about 800%. In the absence of transaction costs this does not influence the portfolio performance. Further research could address the consideration of transaction costs of the regime dependent optimization.

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