ACTIVE PORTFOLIO MANAGEMENT IN THE PRESENCE OF REGIME SWITCHING: WHAT ARE THE BENEFITS OF DEFENSIVE ASSET ALLOCATION STRATEGIES IF THE INVESTOR FACES BEAR MARKETS?

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ABSTRACT. This paper studies the asset allocation decision in the presence of regime switching in stock market returns. The analysis is based on two stock indices: DJI 30 and OMX 30. The two-step optimization procedure employed points towards the usage of defensive asset allocation strategies under bear markets and ordinary index tracking strategies under bull markets. The out-of-sample experiments strengthen the performance of active strategies that distinguish between different regimes. Moreover, the Sharpe ratios of portfolios based on such strategies are higher than the ones of ordinary index tracking based portfolios.

1. INTRODUCTION AND LITERATURE REVIEW

The portfolio selection process has become an important issue of modern portfolio management. The traditional stock selection in prior related studies involves the optimization procedure in a mean-variance framework. Based on the seminal work of Markowitz (1959), Sharpe (1964), Black (1972) and Black and Litterman (1992) proposed a means of estimating expected asset returns to obtain better-behaved portfolio models. The Black and Litterman (1992) model, which is often referred to as active strategy, optimizes expected stock returns in a mean-variance framework, constructing a portfolio in which bets are taken only on stocks for which the portfolio management has opinions on future expected returns. Thereby, the magnitude of bets in relation to the equilibrium portfolio weights depends on the confidence levels specified by the management and on a parameter specifying the weight of the collected investor beliefs in relation to the market equilibrium, the weight-on-views. The Black and Litterman (1992) portfolio optimization model is widely applied, discussed and refined in the literature, as in studies by Chow (1995), Jones et al. (2007), Martellini and Ziemann (2007). However, Phengpis and Swanson (2011) argue that the magnitude of suggested gains that can be realized after portfolio formation is questionable, as the historically optimized portfolio tends to perform poorly out-of-sample due to the estimation error. In particular, the stocks that have performed well tend to be overweighted in the historically optimized portfolio. Other portfolio optimization procedures, which focus mainly on tracking indices, are often referred to as passive strategies. In particular, optimized sampling as suggested by van Montfort, Visser and Fijn van Draat (2008) aims to find the portfolio that has tracked the underlying index as much as possible in the past, hoping it will track the index the same way in future periods.

According to Alexander (1999) and Alexander and Dimitriu (2005a), correlation based portfolios can be very sensitive to the presence of outliers, non-stationarity or volatility clustering.

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Hence, they consider portfolio optimization procedures which are based on cointegration analysis. The cointegration approach to portfolio modeling allows for using the entire information set in a system of stock prices. According to Granger and Terasvirta (1993) stock prices are long-memory processes and therefore, cointegration can explain their long-run behavior. Friesen et al. (2009) argue that there is convincing evidence that stock prices display short-term momentum over periods of six to twelve months involving mean reversion, as already suggested in studies by De Bondt and Thaler (1985), Chopra et al. (1992) and Jegadeesh and Titman (1993). In contrast to correlation analysis, optimization procedures based on cointegration analysis aim at tracking the stochastic trends cached in the stock prices. Overall, this line of research outperforms its counterpart based on correlation analysis. Studies that investigate the performance of portfolio optimization procedures based on cointegration analysis can be found in Alexander and Dimitriu (2005a, b), Grobys (2010) and Phengpis and Swanson (2011).

Even though Alexander and Dimitriu (2005b) conclude that the entire abnormal return of a cointegration based trading strategy is associated with the high volatility regime, their study does not account for an actively selected defensive strategy in such stock market crashes. But what is the advantage of taking actively defensive positions when the investor faces a persistent price bust, respectively, stock market crash? Extreme positions in stocks are basically associated with higher trading costs, but may it be that lowered losses induced by defensive positions overcome the potentially higher trading costs¹ aforementioned? Boldin and Cici (2010) show that less than half of the actively managed equity funds outperform the average S&P 500 index fund, which suggests that more importance should be given to strategically timing the market.

Guidolin and Timmermann (2008) see mounting empirical evidence that asset returns do not follow linear processes with stable coefficients, but a more complicated process involving different regimes which are associated with individual return distributions. The latter is also supported by studies of Ang and Bekaert (2002a, b), Ang and Chen (2002), Guidolin and Timmermann (2005a, b, 2006a, b, c), Perez-Quiros and Timmermann (2001) and Whitelaw (2001). Typically observed regimes in stock markets are often referred to as bull- and bear markets or, in statistical terms, as low frequency trends which switch between persistent periods exhibiting positive or negative returns on expectation. Traditional methods which are employed in order to identify these trends rest typically upon an expost assessment of the stock markets' peaks and troughs. Gonzalez et al. (2005), Lunde and Timmermann (2004) and Pagan and Sossounov (2003) provide such dating algorithms involving a set of rules for classification. These approaches have in common that a turning point can only be figured out several observations after it had occurred. Furthermore, the latent nature of low frequency trends is not accounted for in any of these methodologies. Thus, they do not allow for statistical inference. Probability models which can be employed for statistical inference in the presence of low frequency trends are the Markov-Switching (MS) models for which transitions between states, respectively, regimes are governed by a discrete parameter Markov chain (Guidolin and Timmermann, 2008, Grobys 2011).

Ang and Bekaert (2002a) introduce regime switching into a dynamic international asset allocation setting. Thereby, they investigate a U.S. investor with constant relative risk aversion maximizing the expected end-of-period utility and dynamically rebalancing the portfolio. While estimating regime-switching models on U.S., U.K., and German equity, their findings give evidence of a high-volatility, high-correlation regime which tends to coincide with a bear market. The main result of the study is that the high volatility regime mostly induces a switch toward the lower volatility assets, which are cash (if available), U.S. equity, and also German equity, if available. The statistical model employed by Ang and Bekaert (2002a) is a two-state regime

¹If a portfolio manager takes extreme positions in stocks to take advantage of momentum effects, the positions have to be changed, respectively, rebalanced more frequently in comparison to a portfolio that is constructed for ordinary index replication, only. This may be a matter of the momentum effects, as the latter can be seen as stochastic short-run movements requiring highly frequented rebalancing and, as a consequence, higher trading costs (see also section "Discussion of the Results").

switching model in which the states are assumed to be observable as mentioned by Guidolin and Timmermann (2008).

Guidolin and Timmermann (2008) study the asset allocation decision in the presence of regime switching in asset returns and their model involves four states: crash, slow growth, bull and recovery. In contrast to Ang and Bekaert (2002a), Guidolin and Timmermann (2008) and Grobys (2011) treat the regime switching variable as unobservable. Guidolin and Timmermann (2008) investigate the optimal asset allocation of an US investor between bonds, stocks and cash. Against Barberis' (2000) suggestion that the weight on stocks should increase as a function of the investor's horizon, Guidolin and Timmermann (2008) find that this is no longer the case when change in regimes may occur, as the weight on stocks increases in the investment horizon only when investor faces the crash state at the time when the investment decision is made. Against this, the optimal allocation to stocks declines as a function of the investment horizon when the investors face a bull market, slow growth or recovery state. However, Guidolin and Timmermann (2008) underline that investors are supposed to adjust their portfolio weights as new information arrives.

This contribution takes the presence of stock market regimes as a starting point and proceeds to characterizing asset allocation implications for the equity portfolio management. The modeling approach can be divided in two parts: in the first step, the current stock market regime is estimated and this approach is closely related to Ang and Bekaert (2002a, b) and in particular Guidolin and Timmermann (2008) and Grobys (2011). The second step presents the optimization procedure which is dependent on the current regime. Thereby, the cointegration approach in accordance to Alexander (1999) and Alexander and Dimitriu (2005a, b) is employed in order to exploit stock markets' short-term momentum. There are no studies available that take into account both features at the same time, namely a multiple asset allocation procedure embedded in a two-step approach whereby the optimization procedure is dependent on the current state of the system. This contribution which belongs to the literature of active portfolio management remedies this current gap.

The next section provides an overview about the statistical methodology including statistical tests to assess the model selection. Afterwards, the results of the study are discussed. The last section concludes and identifies possible areas of future research.

2. ECONOMETRIC METHODOLOGY

The section describes a multiple asset allocation strategy with portfolios that aim at tracking the underlying stock indices. Two strategies will be compared with each other. The first strategy will be referred to as ordinary index-tracking strategy and considered as a passive asset allocation strategy where the tracking-portfolio will be rebalanced regularly, irrespective if the investor faces a bull- or bear-market. The second strategy will be referred to as defensive strategy and considered as an active asset allocation strategy where the tracked index switches between the ordinary index and an artificial index. The latter will be referred to as defensive index. In doing so, the current regime can either be a bull-market where the investor expects positive returns in the middle-run, or a bear-market where negative returns are expected.

Following Guidolin and Timmermann (2008), it will be supposed that the stock markets' mean and covariances in returns are driven by a common state variable, S_m , that takes integer values 1, ..., K:

$$r_m = \mu_{S_m} + \sum_{j=1}^p a_{a,S_m} \cdot r_{m-j} + \varepsilon_m \tag{1}$$

$$\begin{pmatrix} r_{1m} \\ \vdots \\ r_{Nm} \end{pmatrix} = \begin{pmatrix} \mu_{1S_m} \\ \vdots \\ \mu_{NS_m} \end{pmatrix} + \sum_{j=1}^p A_{j,S_m} \begin{pmatrix} r_{1m-j} \\ \vdots \\ r_{Nm-j} \end{pmatrix} + \begin{pmatrix} \varepsilon_{1m} \\ \vdots \\ \varepsilon_{Nm} \end{pmatrix}$$
(2)

In equation (1) μ_{S_m} denotes the expectation of the respective stock-market and r_m denotes the corresponding stock-market return at time *m* where the index m = 1, ..., M indicates a monthly frequency series of log-returns. The parameters $a_{1,S_m}, ..., a_{p,S_m}$ and the mean μ_{S_m} depend on the current state S_m . In Equation (2), $(S_{1S_m}, ..., S_{NS_m})'$ is the expectation of the vector of returns $(r_{1_m}, ..., r_{N_m})'$ and it is state-dependent. Furthermore, $(\varepsilon_{1_m}, ..., \varepsilon_{N_m})' \boxdot N(0, \sum_{S_m})$ where N denotes the number of stock markets. If k = 1.0, equation (1) will be in line with Guidolin and Timmermann (2008) simplified to a standard vector-autoregression.

In the following, regime switching in the state variable S_m (i.e. from "bear market" to "bull market" for instance) is governed by the transition probability matrix P that is a K^2 matrix with elements

$$Pr(S_m = i \mid S_{m-1} = j) = p_{ij}, withi, j = 1, ..., K$$
(3)

where K denotes the number of states that are accounted for. For K = 2 the transition probability matrix is

$$P = \begin{pmatrix} p11 & p21\\ p12 & p22 \end{pmatrix} = \begin{pmatrix} p11 & 1-p22\\ 1-p11 & p22 \end{pmatrix} = (pij)$$
(4)

Hence, each regime is the realization of a first-order Markov chain with constant transition probabilities. As the state variable S_m is unobservable, a filtered estimate has to be computed from the vector r_m . Thus, the model allows the return and covariances to vary across states involving strong asset allocation implications for the active asset allocation strategy considered here. For instance, knowing that the current state is a bear state, the management will invest in stocks exhibiting the lowest expected losses and, thus, are expected to exhibit the most defensive properties. Estimation will be performed by maximizing the log-likelihood function associated with (1)-(4), respectively, (2)-(4). As S_m is assumed to be unobservable, it has to be treated as latent variable which requires the EM algorithm described in detail by Hamilton (1989) and discussed further by Guidolin and Timmermann (2005a). In determining the market regime the selection between univariate and multivariate models depends on the correlation between stock markets. The latter is tested as follows: under the null hypothesis the N stock markets considered exhibit no significant correlation. In contrast, the alternative hypothesis is in favor of a multivariate model. Under the null hypothesis, the test statistic λ is asymptotically distributed as

$$\lambda = T \sum_{i=2}^{N} \sum_{l=1}^{i-1} \rho_{il}^2 \square^a \chi_q^2$$
(5)

where q = (N-1)N/2 degrees of freedom while T denotes the number of observations taken into account and ρ_{il} denotes the correlation coefficient with $i \neq l$.

The active asset allocation strategy involves a frequent rebalancing of the stock weights. In the following, the Markov switching model is updated quarterly. Quarterly rebalancing methods are also applied in the study of van Montfort, Visser and Fijn van Draat (2008). If the regime switches, the optimal weight allocation will be re-estimated in accordance to the regime (see equations (6)-(12)). In bull market regimes, however, the asset allocation will be the same as for the ordinary index-tracking portfolio. To hold the transaction costs low, the portfolio weights are re-estimated semi-annually as long as the regime is not switching from a bull to a bear market regime. As the duration of bear market regimes is according to Claessens et al. (2009) empirically shorter in comparison to bull market regimes, the estimated stock weights based on tracking an artificial index are held constant until the regime switches to a bull market again. The latter constraint can also be considered in the light of transaction costs, as a change from an offensive to a defensive allocation and may be associated with high transaction costs due to extreme positions in stocks which is discussed in more detail in Alexander and Dimitriu (2005b).

Following Guidolin and Timmermann (2008) the current regime is estimated with Markov-Switching models while accounting only for data from $\alpha m = 1, ..., M - (R - \omega \cdot \eta)$ where R denotes the overall out-of-sample period in months, ϖ denotes the rebalancing frequency and η the rebalancing time. For instance, if the first state probability forecast is estimated for January 2, 2001 the dataset includes monthly log-return data from the first observation until the latter month. If =3, which corresponds to a quarterly rebalancing strategy, the dataset for the next forecast will include information from the first observation until April 2, 2001 and so on. In this manner, the current estimate will act as forecast of the respective regime that will be taken into account concerning the asset allocation decision. A probability threshold is used as an operational criterion instead of a statistical criterion which is also in line with Alexander and Dimitriu (2005b), who argue that standard out-of sample testing methods are not applicable to Markov-Switching models due to the presence of nuisance parameters. Moreover, the approach regarding the use of the information set is also in line with Guidolin and Timmermann (2008), who mention that the choice of the asset allocation could itself have been benefited from fullsample information, since the approach uses no unavailable data at the time of the estimation. Thus, the defensive strategy is employed if and only if the probability threshold is exceeded.

If the corresponding Markov-Switching model suggests that the investor faces a bear market, the tracked artificial index and is constructed in line with Grobys (2010) as follows. A linear trend term is added to the historical index returns that switches the direction on the day where the local maximum of the time series in price levels is achieved such that

$$p_{\delta,t}^{Index} = c + \sum_{i=1}^{t} R_t^{Index} - \sum_{i=1}^{t} \delta \cdot i$$
(6)

for $t = 1, ..., t^{\max}$

$$p_{\delta,t}^{Index} = p_{\delta,t^{\max}}^{Index} + \sum_{i=1}^{t} R_t^{Index} - \sum_{i=1}^{t} \delta \cdot i$$
(7)

for $t=t^{\max}+1,...,T$

where δ denotes a factor that is subtracted and, respectively, added to the index uniformly distributed over time in daily terms which is, according to Alexander and Dimitriu (2005a) a usual approach to construct enhanced indices. R_t^{Index} denotes the ordinary daily return of the corresponding stock index and $t = t^{\max} + 1, ..., T$ denotes the in-sample data employed in the optimization procedure. It is worth mentioning that the maximum likelihood function employed to estimate the optimal weight allocation accounts for daily frequency data (i.e. the second step of the procedure), whereas the maximum likelihood approach to estimate the current regime accounts for monthly data instead (i.e. the first step of the procedure). Both approaches are usually applied in empirical studies (Guidolin and Timmermann, 2005, Guidolin and Timmermann, 2008, van Montefort, Visser and Fijn van Draat, 2008). Since they operate with integrated time series, both approaches rely on cointegration and are widely used is studies as the ones of Alexander and Dimitriu (2005a,b) and Grobys $(2010)^2$. Equations (6) and (7) involve that the linear trend δ is first subtracted from the market returns and switches the direction at point t^{max} . Subtracting a linear trend term until t^{max} and adding the term from the local maximum onwards results in an artificial index that is below the corresponding stock index until t^{max} and exhibiting higher returns as the underlying index from t^{max} onwards as the bubble (i.e. the peak of the preceding bull market) disperses (Figure 1). Furthermore,

²However, Alexander (1999) and Alexander and Dimitriu (2005a, b) use an OLS-regression from the logstock prices on the stock index in logs in order to replicate the S&P 500 index (Alexander and Dimitriu 2005a), whereas Grobys (2010) estimates a restricted maximum-likelihood function. Under Gaussian assumption, the estimators should be similar.

the integrated time series of the stocks employed to track the artificial indices are in line with Grobys (2010) given by

$$p_{h,t} = c + \sum_{i=1}^{T} R_{h,t}$$
 (8)

where $R_{h,t}$ denotes the ordinary daily return of stock h at time t and c is a constant term where $c \in \Box_+$ is chosen such that $p_{h,t} > 0 \forall h = 1, ..., H$ and t = 1, ..., t. Then, the log-likelihood function used to estimate the optimal weight allocation that is assumed to exhibit defensive properties within the out-of-sample period is given by

$$\ln L(\theta, y, \delta) = -\frac{T}{2} \log(2\pi) - \frac{T}{2} \log \sigma^2 - \frac{1}{2} \sum_{t \in T} (\frac{\varepsilon_{\delta, t}^2}{\sigma^2})$$
(9)

where $\varepsilon_{\delta,t}^2 = p_{\delta,t}^{Index} - \sum_{h=1}^{H} \alpha_{h,t} \cdot p_{h,t}$ and $\theta = \{\mu; \sigma^2\}$. Following Alexander and Dimitriu (2005a) and van Montfort, Visser and Fijn van Draat (2008), it is usual to impose restrictions. In the

following though it will be assumed to be sufficient to restrict the weights to sum up to one and to be positive (i.e. prohibition of short selling) which is given by

$$\alpha_{h,t} > \mathbf{0} \tag{10}$$

for h = 1, ..., H

$$\sum_{h=1}^{H} \alpha_{h,t} = 1 \tag{11}$$

The estimation procedure that is associated with defensive strategies may require allocating high weights to stocks which exhibit defensive properties. Therefore, the only restriction which is of importance is the positivity restriction concerning the weights. Once estimated, the weights $\hat{a}_{h,t}$ are held constant as long as the investor faces a bear market regime which is examined quarterly.

Once a bull market regime is ascertained equations (6) and (7) will be substituted by equation (12) because the index which is tracked rests simply upon the integrated time series given by

$$p_t^{Index} = c + \sum_{i=1}^t R_t^{Index}$$
(12)

As long as the Markov-Switching model does not suggest a change of the regime (i.e. from the current bull to bear market regime), the portfolio will be rebalanced semi-annually while taking into account equations (8)-(10). In comparison to the active asset allocation strategy described by equations (1)-(12), the strategy which does not account for equations (1)-(7) will be referred to as passive asset allocation strategy and acts as a benchmark when the models' performances are compared. The passive strategy is rebalanced semi-annually only, irrespective if the investor faces a bull- or bear market regime. However, the passive strategy is supposed to be associated with lower transaction costs. The investor may expect higher transaction costs as the position he takes becomes more defensive, that is, the chosen factor δ is larger.

3. THE DATA

The analysis is based on two stock markets (i.e. N=2), the American and the European one, represented by the DJI 30 and the OMX 30, respectively. The source of the data is a cost free one: www.finance.yahoo.com and www.nasdaqomxnordic.com. These indices are also

considered in the multivariate 2-State-Markov-Switching model in Grobys' (2011) study³. In line with Guidolin and Timmermann (2008), monthly (i.e denoted by m) stock market data (i.e. in log-returns) is employed to estimate the 2-State-Markov-Switching. 171 Monthly observation from November 3, 1986 to January 2, 2001 could be employed in order to forecast the current regime on January, 2001. The forecast concerning the next quarter though accounts for data until April 2, 2001 and, hence, includes 174 monthly observations and so on. The regime forecasts are repeated on a quarterly base⁴. For instance, if the model suggests in the first step a bear market regime like on April 2, 2001 (see equations (1)-(4)), the optimization procedure takes into account equations (6)-(7) in the second step. Since the market regime is estimated to switch to a bull market regime on July 2, 2001, equation (12) is taken into account in the optimization procedure (Table 1). However, if the investor had been situated in a bull market in both times, equation (12) would have been employed on April 2, 2001 and the weights would

4. DISCUSSION OF THE RESULTS

have not been updated on July 2, 2001 because the investor rebalances the portfolio weights

only every second quarter as long as he/she faces a bull market.

The models are estimated for K = 2, where k = 1 denotes the bull state and k = 2 denotes the bear state. According to the HQ-and SC-criterion, the lag order is p = 0, which is in line with the common finding that stock market returns of developed countries do not exhibit patterns of autocorrelation⁵. The statistical test for contemporaneous correlation uses 10 years of daily frequency data running from January 2, 1991 until December 29, 2000 corresponding to 2457 observations. This time window used to estimate the correlation covers 10 years of the insample window in daily terms. The correlation between the DJI 30 and the OMX 30 log-returns is estimated to be $\rho_{OMX,DJI} = 0.2910$. Hence, the test statistic $\lambda = 208.02$ (p-value 0.0000) shows that the null hypothesis is clearly rejected. Due to the significant correlation between DJI 30 and OMX 30, a bivariate 2-State-Markov-Switching model is used. Consequently, the current regime is estimated simultaneously on a quarterly base (i.e. $\varpi = 3$ and $\eta = 40, \dots, 1$), beginning on January 2, 2001. The out-of-sample time window runs from January 2, 2001 to January 3, 2011 (i.e. R = 120). While the covariance-matrices are assumed to be constant during each regime, the 2-State-Markov-Switching model transforms contemporaneous correlation into timevarying covariances, as the regimes are dependent on the time m. Equations (13)-(17) show the estimates of the 2-State-Markov model concerning equations (2)-(4) (standard errors are given in parenthesis) and taking into account the overall sample (i.e. November 3, 1986 – January 3, 2011):

$$\left(\frac{\mu_{DJI30,S_1}}{\mu_{OMX30,S_1}}\right) = \left(\frac{\begin{array}{c}0.0062\\(0.0010)\\\hline\\0.0092\\(0.0017)\end{array}\right) + \left(\frac{\varepsilon_{DJI30,m}}{\varepsilon_{OMX30,m}}\right)$$
(13)

for S_1

 $^{^{3}}$ From an economical point of view it can be assessed that 3.34% of Sweden's imported goods in 2010 were produced in the USA, whereas in the corresponding period 7.29% of all produced goods in Sweden were exported to the USA. Consequently, the USA is apart from Germany, Norway and the United Kingdom one of Sweden's largest trade partners (see www.scb.se). In the present study it is assumed that such interactions are also embedded in a simultaneous movement of the economies' stock indices, which is sometimes referred to as international stock market integration.

⁴The strategy assumes that regimes are persistent. If the expected duration of each regime is longer than the update of the current regime estimate (i.e. every quarter), the investor assumes here that the regime is not changing until the next update.

⁵The Hannan-Quinn (HQ) and Schwarz (SC) criterion are selection criteria concerning the optimal lag-order in a VAR- model.

$$\left(\frac{\mu_{DJI30,S_1}}{\mu_{OMX30,S_1}}\right) = \left(\frac{-0.0066}{(0.0034)} - 0.0127 \\ (0.0051) \end{array}\right) + \left(\frac{\varepsilon_{DJI30,m}}{\varepsilon_{OMX30,m}}\right)$$
(14)

for S_2

with
$$\sum_{S_1} = \begin{pmatrix} 1.90e - 04 & -1.00e - 05\\ (2.00e - 05) & (2.00e - 05)\\ -1.00e - 05 & 4.30e - 04\\ (2.00e - 05) & (2.00e - 05) \end{pmatrix}$$
 (15)

for S_1

and
$$\sum_{S_2} = \begin{pmatrix} 8.50e - 04 & 4.00e - 05\\ (1.30e - 04) & (1.50e - 04)\\ 4.00e - 05 & 1.79e - 03\\ (1.50e - 04) & (3.10e - 04) \end{pmatrix}$$
 (16)

and
$$S_m = \begin{pmatrix} 0.94 & 0.17 \\ (0.05) & (0.06) \\ 0.06 & 0.83 \\ (0.02) & (0.04) \end{pmatrix}$$
 (17)

The expected duration of the bull market regime is estimated at 16.53 months, whereas the corresponding figure concerning the bear market regime is estimated at 5.86 months. The covariance matrix in the Markov-Switching model is dependent on the current regime and, hence, time-varying. Equations (15) and (16) show that the monthly covariance is estimated to be negative across these stock markets during bull states and positive during bear states. Estimating the quarterly updated (i.e. a rolling time window and $\varpi = 3$) 2-State-Markov model from January 2, 2001 onwards suggests bear-market forecasts as given in figure 2. Figure 2 shows the estimated regime including the whole sample as given by equations (12) - (14) and the forecasted bear market regime where only information until time $M - (R - \varpi \cdot \eta)$ is taken into account. A probability threshold of 0.90 implies that the defensive strategy is applied only if the bear-market state probability forecast in the current quarter exceeds the threshold. Table 1 shows the asset allocation suggested by this approach for the out-of-sample period. The forecast covers ten years, January 2, 2001 - January 3, 2011. To estimate the maximum likelihood function concerning equations (4)-(10), 750 days of daily frequency data is employed which is in line with Alexander and Dimitriu (2005a). The active strategy indicates a defensive weight allocation strategy for April 2, 2001, October 1, 2001, July 1, 2002 and July 1, 2008 (see table 1 in association with figure 2) and five different portfolios will be estimated for both stock markets. Thereby, the factor δ varies between those estimated portfolio weight allocations where $\delta \in \{0.04, 0.08, 0.12, 0.16, 0.20\}$. Thus, portfolio 1 (i.e. for each stock market) accounts for $\delta_1 = 0.04$ which corresponds to the enhancing factor being added, respectively, subtracted (see equations (6) and (7)) by 10% in annual terms.

Analogously, the enhancement factor $\delta_2 = 0.08$ of portfolio 2 corresponds to an active asset allocation strategy, where the artificial index tracked deviates 20% from the ordinary index and so on (Figure 1).Table 2 shows that defensive asset allocation strategies which suggest deviations from the ordinary index between 40%-50% performed the best concerning strategies related to the DJI 30. All actively managed portfolios (i.e. portfolio 1 – portfolio 5) outperform the benchmark which is portfolio 0 (Figure 3). The latter is an ordinary index tracking portfolio where the weights are re-estimated semi-annually, irrespective of the current regime. However, this passive asset allocation strategy still dominates the stock index as the Sharpe ratio (i.e. 0.28) is twice as much as the DJI 30's Sharpe ratio which was 0.14 within the overall out-of-sample period.

However, the results differ concerning the Swedish stock market. The higher the deviation the lower the Sharpe ratios when the whole out-of-sample period is considered. Here, the ordinary index tracking portfolio (i.e. portfolio 0) dominates all active asset allocation strategies as well as the index as its Sharpe ratio of 0.20 is higher in comparison to actively managed portfolios⁶.

In order to determine if the latter outcome can be traced back to the active strategy itself or if this outcome is rather a fact of dataset limitations concerning the dataset of stocks, a sub-sample period will be investigated. The 2-State-Markov model suggests the latest bear market from July 1, 2008 – April 1, 2009 as shown in figure 2 and table 1. As the equity prices began to fall already before July 1, 2008, a sample including data from October 1, 2007-March 31, 2009 will be considered. Consequently, this time window also covers the financial crisis period in 2008. Table 3 shows that the DJI 30 had a return of -9.33% p.a. within this period, whereas the OMX 30 exhibited a return of -8.33%. Considering the US-stock market, portfolios 4 and 5 dominate the benchmark portfolio as the increase in return being 75.68% corresponds to a marginal increase of 5.26% in volatility only (the corresponding figures concerning portfolio 5 are 76.62%increase in returns associated with 7.43% increase in volatility). Considering the Swedish stock market, the benchmark portfolio exhibits a return of -7.32% p.a. with an annual volatility of 15.43%. However, portfolio 2 exhibits 24.70% higher annual returns, associated with an increase of 16.01% in volatility and thus dominates the benchmark portfolio⁷. Consequently, the reason for differences concerning the strategies performances given different stock indices can be traced back to dataset limitations since it was not possible to replicate stochastic processes such as given by the defensive artificial indices being tracked. In particular, the period April 2, 2001 – April 1, 2003 shows an underperformance of these defensive asset allocation strategies (see figure 2). The underperformance of the defensive strategies concerning the Swedish stock market can be attributed to both the bias regarding the preselected stocks (i.e. only 17 of 30 stock could be accounted for) and the reliability of the forecasted regime since the regime-switching model's forecasted regimes exhibited lower deviations from the realized regimes during the second part of the out-of-sample window (i.e. 31% deviation on average during Jan 2001-Dec 2005 and 14%deviation during Jan 2006-Dec 2010). Furthermore, tables 2 and 3 show that the trading costs increase as a function of the trading volume. The more defensive the taken positions in stocks, the higher the trading volume and, as a consequence, the higher the trading costs.

Although the common academic literature predominately takes large capitalization stock indices into account in the context of empirical stock market analyses, it could be shown here that also smaller European stock indices such as the Swedish stock index OMX 30 switches contemporaneously with the US-stock market from bull to bear markets and vice versa. Thus, the studies of Ang and Baekert (2002a) can be supported. However, the bivariate model may account for any combination of stock markets that are highly correlated such as the German's leading index DAX 30, and the DJI 30 or the British's leading index FTSE 100 and the DJI 30 or DAX 30. The bivariate DJI 30 – OMX 30 model is selected for illustration purposes and in order to stand out from common studies. Apart from accounting for time-varying correlations between international stock markets, under the bivariate setup the states are more persistent for the US stock market (Table 4). However, the bivariate model does not determine which index drives the variable that initiates switches of the states even though it may be assumed that the US-stock index involves the hidden factor. This may be subject to future research.

The operational criterion suggests at four points of time a bear market which implies a defensive asset allocation (Table 1). The bear market regime in the wake of the financial crisis

 $^{^{6}}$ Note that the 29 out of 30 stocks are accounted for when running the maximum likelihood function concerning the DJI 30 index. Due to data set limitations though 17 out of 30 stocks could be taken into account only when running the optimization procedure with respect to the OMX 30 as the access to stock data becomes the more limited the further away the historical selected data.

⁷These results hold even if the net returns (i.e. after transaction costs) are taken into account.

is estimated by far be more persistent as the stock market crash of 2001-2002. In contrast to Alexander and Dimitriu (2005a, b), the ordinary cointegration portfolio is employed as benchmark in order to analyze the performance differences.

In contrast to Guidolion and Timmermann (2008) in this study only two states (i.e. bull and bear market) are taken into account which is also in line with Ang and Baekert (2002a), as active portfolio management typically differentiates only between offensive and defensive strategies where defensive strategies are employed when market participants face bear markets. The present study suggests an ordinary index tracking strategy based on cointegration if the management faces a bull market, whereas defensive strategies are employed in case of a bear market. However, offensive strategies may also involve enhanced index tracking strategies as suggested by Alexander and Dimitriu (2005a) or Grobys (2010). This could even improve the overall index tracking portfolio's performance. Alexander and Dimitriu (2005a) point out that tracking the artificial indices by more than plus/minus 5% does not result in an equity portfolio with higher Sharpe ratios as the portfolio's volatility increases and the abnormal returns become insignificant. This outcome cannot be supported in this study which takes into account the market regime: Considering the US-stock market, table 2 shows that the Sharpe ratios increase as δ increases (see also figure 4). The latter can be seen as market anomalies which may appear due to overreactions during bear market regimes.

Furthermore, this study employs monthly stock market data for estimating the current market regime (i.e. step one in the procedure). Such data frequency is also used in the 4-State Markov-Switching framework suggested by Guidolion and Timmermann (2008). However, the data set contains, due to data set limitations, fewer observations (i.e. 292 monthly observations) in comparison to the study of Guidolion and Timmermann (2008), who account for 552 monthly observations. As 14 parameters are estimated only, the data set limitations do not delimitate the parameter estimation results. The estimated parameters are clearly significant as provided by equations (13)-(17).

The second step of the optimization procedure, which estimates the optimal weight allocation of the artificial index, uses linearized stock prices. It is worth observing that no excess returns are gained if either ordinary returns, log-returns or log-prices are employed in the maximum-likelihood function. This shows that linearized prices, as suggested by Grobys (2010), are a useful tool in order to cache assets' short term momentum (see De Bondt and Thaler, 1985, Chopra et al., 1992 and Jegadeesh and Titman, 1993).

Ang and Baekert's (2002a) findings that United States and the United Kingdom face the same regime shifts generated by the benchmark regime-switching model can be supported in the sense that the Swedish stock market and the US-stock market are driven by the same stochastic variable into bull and bear market regimes. Another outcome of Ang and Baekert (2002a), namely that in one regime the equity returns exhibit a lower conditional mean, much higher volatility, and are more highly correlated compared to the other regime, can be supported as well (see equations (13)-(17)). In contrast to Ang and Bekaert (2002a) though, in this study the regime switching variable is treated as unobservable which is in line with Guidolin and Timmermann (2008), Alexander and Dimitriu (2005b) and Grobys (2011).

5. CONCLUSION

Accounting for actively managed defensive strategies enhances the gains of the equity portfolio. Even though passive strategies suggest an implicit market timing factor due to an equilibrium effect (Alexander and Dimitriu, 2005b), the present study shows that when taking into account different regimes active strategies perform better. The model for the US stock market clearly shows that in bear markets the actively managed equity portfolios outperform the common cointegration based portfolio by successfully capturing the stochastic short-run trends. The maximum-likelihood function that is constructed to extract stock prices following contrary trends to the index, pitches on stocks that exhibit defensive movements. For instance on July 1, 2008 portfolio 5 (i.e. concerning the US-market) suggests an asset allocation of 100% to the stock "The home Depot, Inc." which belongs to the furniture selling industry. The higher the deviation of the artificial index selected, the more weight is allocated to this stock⁸. Similar patterns can be investigated on the Swedish stock market and the stock company "Securitas" which provides security services. However, there remains demand for future research to determine the linkages between the asset allocation which is estimated by this maximum-likelihood procedure and the stock price dispersion. Furthermore, the Markov-Switching model could account for three states, for instance, where the bull- and bear market state is extended by a state capturing a potential sideward movement of the index. The actively managed portfolio may differentiate between strategies such as enhanced index tracking if the Markov-Switching model predicts a bull market, defensive index tracking strategies if the investor faces a bear market and an ordinary index tracking strategy otherwise.

References

- Ang A., & Bekaert, G. (2002a). International Asset Allocation With Regime Shifts. Review of Financial Studies (15)/4, pp. 1137-1187.
- [2] Ang, A. & Bekaert, G. (2002b). Regime Switches In Interest Rates. Journal of Business and Economic Statistics (20), pp. 163-182.
- [3] Ang A., & Chen, J. (2002). Asymmetric Correlations Of Equity Portfolios. Journal of Financial Economics (63), pp. 443-494.
- [4] Alexander C. (1999). Optimal Hedging Using Cointegration. Philosophical Transactions of the Royal Society Series A, 357, pp. 2039-2058.
- [5] Alexander, C. & Dimitriu, A. (2005a). Indexing And Statistical Arbitrage. The Journal of Portfolio Management (31)/2, 50-63.
- [6] Alexander, C. & Dimitriu, A. (2005b). Indexing, Cointegration And Equity Market Regimes. International Journal of Finance and Economics (10), pp.213-231.
- [7] Barberis, N. (2000). Investing For The Long Run When Returns Are Predictable, Journal of Finance (55), pp. 225-264.
- [8] Black, F. (1972). Capital Market Equilibrium With Restricted Borrowing. Journal of Business (45), pp. 444-454.
- [9] Black, F., & Litterman, R. (1992). Global Portfolio Optimization, Financial Analysts Journal (48), pp. 28-43.
- [10] Boldin, M., & Cici, G. (2010). The Index Rationality Paradox. Journal of Banking and Finance (34), pp. 33-43.
- [11] Claessens, S., Kose, M.A., & Terrones, M.E. (2009). What Happens During Recessions, Crunches And Busts? Economic Policy (60), pp.653-700.
- [12] Chow, G. (1995). Portfolio Selection Based On Return, Risk, And Relative Performance Financial Analysts Journal (51)/ 2, pp. 54-60.
- [13] Chopra, N., Lakonishok, J., & Ritter, J., (1992). Measuring Abnormal Performance. Do Stocks Overreact? Journal of Financial Economics (31), pp. 235—268.
- [14] De Bondt, W., & Thaler, R., (1985). Does The Stock Market Overreact? Journal of Finance (40), pp. 793-805.
- [15] Friesen, G.C., Weller, P.A., & Dunham, L.M. (2009). Price Trends And Patterns In Technical Analysis: A Theoretical And Empirical Examination. Journal of Banking & Finance (33), pp. 1089-1100.
- [16] Gonzalez, L., Powell, J. G., Shi, J., & A. Wilson (2005). Two Centuries of Bull And Bear Market Cycles. International Review of Economics and Finance (14), pp. 469—486.
- [17] Granger, C.W.J., & Terasvirta, T. (1993). Modelling Nonlinear Economic Relationships, Chapter 5. Oxford: Oxford University Press.
- [18] Grobys, K. (2010). Do Business Cycles Exhibit Beneficial Information For Portfolio Management? An Empirical Application Of Statistical Arbitrage. The Review of Finance and Banking (2)/2, pp. 41-56.
- [19] Grobys, K. (2011). Are Different National Stock Markets Driven By The Same Stochastic Hidden Variable? The Review of Finance and Banking (3)/1, pp. 21-30.
- [20] Guidolin, M., & Timmermann, A. (2005a). Strategic Asset Allocation And Consumption Decisions Under Multivariate Regime Switching, Federal Reserve Bank of St. Louis working paper No. 2005-002A.
- [21] Guidolin, M., & Timmermann, A. (2005b). Size And Value Anomalies Under Regime Switching. Federal Reserve Bank of St. Louis working paper No. 2005-007A.

⁸The weight allocation vector July 1, 2008 concerning the stock "The Home Depot, Inc." for the portfolios $(0, 1, \ldots, 5)$ is given by (6.85%, 17.92%, 59.56%, 82.61%, 94.83%, 100%). At this time, the Markov-Switching model predicts a bear market with probability > 90% and consequently, the weights are held constant as long as the regime is not switching (i.e. 3 quarters in this case).

- [22] Guidolin, M., & Timmermann, A. (2006a). An Econometric Model Of Nonlinear Dynamics In The Joint Distribution Of Stock And Bond Returns. Journal of Applied Econometrics (21), pp.1-22.
- [23] Guidolin, M., & Timmermann, A. (2006b). Term Structure Of Risk Under Alternative Econometric Specifications. Journal of Econometrics (131), pp. 285-308.
- [24] Guidolin, M., & Timmermann, A. (2006c). International Asset Allocation Under Regime Switching, Skew And Kurtosis Preferences. Federal Reserve Bank of St. Louis working paper No. 2005-018A.
- [25] Guidolin, M., & Timmermann, A. (2008). Asset Allocation Under Multivariate Regime Switching. Journal of Economic Dynamics and Control (31), pp. 3503-3544.
- [26] Hamilton, J. (1989). A New Approach To The Economic Analysis Of Nonstationary Time Series And The Business Cycle. Econometrica (57), pp. 357-384.
- [27] Jegadeesh, N., & Titman, S., (1993). Returns To Buying Winners And Selling Losers: Implications For Stock Market Efficiency. Journal of Finance (48), pp. 65—91.
- [28] Jones, R.C., Lim, T., & Zangari, P.J. (2007). The Black-Litterman Model For Structured Equity Portfolios. The Journal of Portfolio Management (33)/2, pp. 24-33.
- [29] Lunde, A., & Timmermann, A. G., (2004). Duration Dependence In Stock Prices: An Analysis Of Bull And Bear Markets. Journal of Business & Economic Statistics (22)/3, pp.253—273.
- [30] Markowitz, H. (1959). Portfolio Selection: Efficient Diversification of Investments. New York: John Wiley & Sons.
- [31] Martellini, L., & Ziemann, V. (2007). Extending Black-Litterman Analysis Beyond The Mean-Variance Framework. The Journal of Portfolio Management (33)/4, pp. 33-44.
- [32] Montfort, K., Visser, E., & Fijn van Draat, L. (2008). Index Tracking By Means Of Optimized Sampling. Journal of Portfolio Management (34), pp.143-151.
- [33] Sharpe, W. (1964). Capital Asset Prices: A Theory Of Market Equilibrium Under Conditions Of Risk. Journal of Finance, pp, 425-442.
- [34] Pagan, A. R., & Sossounov, K. A. (2003). A Simple Framework For Analyzing Bull And Bear Markets. Journal of Applied Econometrics (18)/1, pp. 23—46.
- [35] Perez-Quiros, G. & Timmermann, A. (2001). Business Cycle Asymmetries In Stock Returns: Evidence From Higher Order Moments And Conditional Densities, Journal of Econometrics (103), pp. 259-306.
- [36] Phengpis, C., & Swanson, P.E. (2011). Optimization, Cointegration And Diversification Gains From International Portfolios: An Out-Of-Sample Analysis. Review of Quantitative Finance and Accounting (36)/2, pp.269-286.
- [37] Van Montfort, K., Visser, E., & Fijn van Draat, L. (2008). Index Tracking By Means Of Optimized Sampling. The Journal of Portfolio Management (34)/2, pp. 143-152.
- [38] Whitelaw, R. (2001). Stock Market Risk And Return: An Equilibrium Approach. Review of Financial Studies (13), pp. 521-548.



^{6.} Appendix



Note: 750 daily observations are taken into account where the local maximum was achieved on October 9, 2007 where the DJI 30 quoted 14.165 points.



Figure 2: Bear-regime forecasts of the multivariate 2-State-Markov-Model



Figure 3: Defensive portfolios and the OMX 30 in the out-of-sample period

Figure 4: Defensive portfolios and the DJI 30 in the out-of-sample period



Date	Action	Date	Action
2-Jan-2001	1	3-Jan-2006	3
2-Apr-2001	2	3-Apr-2006	1
2-Jul-2001	1	3-Jul-2006	3
1-Oct-2001	2	2-Oct-2006	1
2-Jan-2002	1	3-Jan-2007	3
1-Apr-2002	3	2-Apr-2007	1
1-Jul-2002	2	2-Jul-2007	3
1-Oct-2002	4	1-Oct-2007	1
2-Jan-2003	4	2-Jan-2008	3
1-Apr-2003	1	1-Apr-2008	1
1-Jul-2003	3	1-Jul-2008	2
1-Oct-2003	1	1-Oct-2008	4
2-Jan-2004	3	2-Jan-2009	4
1-Apr-2004	1	1-Apr-2009	1
1-Jul-2004	3	1-Jul-2009	3
1-Oct-2004	1	1-Oct-2009	1
3-Jan-2005	3	4-Jan-2010	3
1-Apr-2005	1	1-Apr-2010	1
1-Jul-2005	3	1-Jul-2010	3
3-Oct-2005	1	1-Oct-2010	1

 Table 1: Active weight allocation strategies suggested by the Markov-Switching

 model

Asset allocation strategies:

1. The weight allocation being selected is an ordinary index-tracking strategy.

2. The weight allocation being selected is in accordance to a defensive strategy.

3. The weight allocation being selected is the ordinary index-tracking strategy which is estimated in the previous period (i.e. the weights are held constant).

4. The weight allocation being selected is the defensive weight allocation strategy which is estimated in the previous period (i.e. the weights are held constant).

Table 2: Statistical properties and performances in the out-of-sample periodBenchmarkDJI 30

Dencimark	D11 20						
Asset	DJI 30	Portfolio	Portfolio	Portfolio	Portfolio	Portfolio	Portfolio
		0	1	2	3	4	5
Gross re-	$2,\!38\%$	6,20%	$6{,}65\%$	$8,\!67\%$	8,97%	$9{,}51\%$	$9,\!45\%$
turn p.a.							
Volatility	$17,\!11\%$	$19,\!61\%$	$18,\!58\%$	$16,\!68\%$	$16,\!15\%$	$16,\!48\%$	$16,\!37\%$
p.a.							
Sharpe ra-	0,14	0,32	$0,\!36$	0,52	$0,\!56$	$0,\!58$	$0,\!58$
tio (before							
$\mathbf{costs})$							
Transaction	-	87%	120%	171%	190%	197%	199%
volume p.a.							
Trading	-	0,70%	0,96%	1,36%	1,52%	1,58%	1,59%
costs p.a.							
Net return	$2,\!38\%$	$5{,}50\%$	$5{,}69\%$	$7,\!30\%$	$7,\!45\%$	$7{,}93\%$	$7,\!86\%$
p.a.							
Sharpe ra-	$0,\!14$	0,28	0,31	0,44	$0,\!46$	$0,\!48$	$0,\!48$
tio (after							
$\mathbf{costs})$							

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Benchmark	OMX 30	OMX 30					
Asset	OMX	Portfolio	Portfolio	Portfolio	Portfolio	Portfolio	Portfolio
	30	0	1	2	3	4	5
Gross re-	$4,\!63\%$	6,72%	$5{,}88\%$	$6,\!12\%$	$5{,}36\%$	$3{,}88\%$	$2,\!64\%$
turn p.a.							
Volatility	$24,\!31\%$	$30,\!10\%$	28,94%	$29,\!87\%$	29,77%	$28,\!80\%$	27,75%
p.a.							
Sharpe ra-	$0,\!19$	0,22	0,20	0,20	0,18	$0,\!13$	0,10
tio (before							
costs)							
Transaction	-	79%	106%	140%	171%	188%	202%
volume p.a.							
Trading	-	$0,\!63\%$	0,85%	$1,\!12\%$	$1,\!36\%$	1,51%	$1,\!61\%$
costs p.a.							
Net return	$4,\!63\%$	6,09%	$5{,}03\%$	$5,\!00\%$	4,00%	$2,\!37\%$	$1,\!03\%$
p.a.							
Sharpe ra-	0,19	0,20	$0,\!17$	$0,\!17$	$0,\!13$	0,08	0,04
tio (after							
costs)							

*Assuming 0.60% per 100% trading volume.

Table 3:	Statistical	properties an	d performances
(during the	financial crise	s period

Benchmark	DJI 30						
Asset	DJI 30	Portfolio	Portfolio	Portfolio	Portfolio	Portfolio	Portfolio
		0	1	2	3	4	5
Gross re-	-9,33%	-9,54%	-8,22%	-3,81%	-2,80%	-2,32%	-2,23%
turn p.a.							
Volatility	10,58%	$14,\!27\%$	$12,\!08\%$	$11,\!31\%$	$13,\!67\%$	$15,\!02\%$	$15{,}33\%$
p.a.							
Sharpe ra-	-0,88	-0,69	-0,68	-0,34	-0,20	-0,17	-0,15
tio (before							
costs)							
Transaction	-	87%	124%	163%	167%	176%	178%
volume p.a.							
Trading	-	0,52%	0,74%	$0,\!98\%$	1,00%	1,06%	$1,\!07\%$
costs p.a.							
Net return	-9,33%	-10,06%	-8,96%	-4,78%	-3,80%	-3,38%	-3,30%
p.a.							
Sharpe ra-	-0,88	-0,70	-0,74	-0,42	-0,28	-0,23	-0,22
tio (after							
costs $)$							

30

Benchmark	OMX 30						
Asset	OMX	Portfolio	Portfolio	Portfolio	Portfolio	Portfolio	Portfolio
	30	0	1	2	3	4	5
Gross re-	-8,12%	-7,32%	-5,66%	-5,34%	$-5,\!64\%$	-6,38%	-5,98%
turn p.a.							
Volatility	$11,\!23\%$	$15,\!43\%$	17,11%	17,90%	$18,\!90\%$	$21,\!53\%$	21,78%
p.a.							
Sharpe ra-	-0,72	-0,47	-0,33	-0,30	-0,30	-0,30	-0,27
tio (before							
costs)							
Transaction	-	119%	117%	139%	155%	180%	197%
volume p.a.							
Trading	-	0,71%	0,70%	0,83%	0,93%	$1,\!08\%$	$1,\!18\%$
costs p.a.							
Net return	-8,12%	-8,03%	-6,36%	-6,17%	-6,57%	-7,46%	-7,16%
p.a.							
Sharpe ra-	-0,88	-0,52	-0,37	-0,34	-0,35	-0,35	-0,33
tio (after							
costs)							

*Assuming 0.60% per 100% trading volume.

Note: The period being considered here is October 1, 2007-March 31, 2009.

Table 4: Univariate MS-models estimates for the DJI 30 and OMX 30 concerning the sample from November 3, 1986 – January 3, 2011*

Stock	State-matrix	Expected duration	Expected duration
Index		of Bull-market	of Bear-market
DJI 30	$S_m^{DJI} = \begin{pmatrix} 0.95 & 0.30\\ (0.06) & (0.12)\\ 0.05 & 0.70\\ (0.02) & (0.09) \end{pmatrix}$	18.63 months	3.38 months
OMX 30	$S_m^{OMX} = \begin{pmatrix} 0.95 & 0.08\\ (0.06) & (0.04)\\ 0.05 & 0.92\\ (0.03) & (0.05) \end{pmatrix}$	19.53 months	13.22 months

* standard-error in parenthesis.

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