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VALUE AT RISK ESTIMATION FOR NON-GAUSSIAN DISTRIBUTIONS

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Abstract. This paper presents a methodology for computing Value at Risk for financial assets that does not follow a normal distribution of return. A back-testing approach have been applied in order to select the best theoretical non-Gaussian distributions that can explain the behavior of the empirical data. In this study, Cauchy, Laplace, Logistic and Beta distributions have been considered. As benchmark, historical distribution, and Extreme Value Theory (EVT) method have been used. The experiment suggests differences in estimation of over 5 times between one method and another.

1. Introduction

The efficient measurement of market risk has become crucial given the significant increase of uncertainty in financial markets. The extensive movements of the prices of the financial assets and the intensive use of the derivative instruments determined the need of risk measures capable of capturing and mitigating the increasingly pronounced market risks. This paper delves into the heart of market risk — the unanticipated volatility of returns on a portfolio, spurred by shifts in stock prices, exchange rates, interest rates, and commodity prices. At the core of this volatility lie the "stylized facts" of financial assets, a term coined by Cont (2001) to describe the empirical regularities in financial market data. Among these is the concept of "heavy tails," indicative of a greater likelihood of extreme variations in asset returns than predicted by the normal distribution. This paper explicitly addresses the challenge of measuring risk in the presence of such heavy-tailed, non-Gaussian distributions.

Contrasting with the Modigliani-Miller theorem of 1958, which posits the irrelevance of risk management in a perfect market, Bartram (2000) underscored the efficacy of risk management strategies in the face of market imperfections such as trading costs, asymmetric information, and tax disparities. Hence, the application of quantitative measures of market risk has become a cornerstone for decision-making processes within financial institutions including banks, investment funds, pension schemes, and hedge funds, as well as non-financial entities.

The preeminent measure in this domain is Value at Risk (VaR), though its calculation remains a subject of debate, lacking a unified methodology (Thompson & McCarthy, 2008). VaR estimates the maximum loss a portfolio might suffer within a specific time frame, under a given probability — typically 95% or 99%. Despite its roots in Markowitz's portfolio theory, VaR represents a significant departure in several ways, as noted by Dowd (2005):

VaR quantifies risk in terms of potential loss magnitude, whereas portfolio theory does so via standard deviation;

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VaR is adaptable to various types of distribution, in contrast to the normal distribution assumption of portfolio theory;

VaR is applicable across diverse risk types, not just market risk.

Originating at J.P. Morgan in the late 1980s, VaR has since established itself as an industry benchmark. However, the literature continues to critically assess VaR models, aiming to refine their accuracy and predictive capability. Beder (1995) demonstrated the variability in VaR estimates across different models applied to identical portfolios, revealing the sensitivity of VaR to the chosen data, assumptions, and methodologies. In his conclusions, VaR does not guarantee a good risk measure, the results varied even fourteen times for the same portfolio.

Küsters, Mittnik, and Paolella (2006) highlight the imperative to evolve VaR models that can reliably predict risk in alignment with emergent regulatory demands. Their comparative study of conditional and unconditional VaR models, when applied to the NASDAQ index, reveals a consistent underestimation of market risks, casting doubt on the models' precision.

Further scrutiny by Giamouridis and Ntoula (2009) compares the effectiveness of historical and parametric approaches, extending their investigation to include both normal distributions and Cornish-Fisher expansions, as well as the generalized Pareto distribution. Their findings indicate that distributions accommodating the heavy tails and asymmetry of financial returns offer superior performance over traditional Gaussian and historical models, especially at the critical 1% threshold. At a less stringent 5% level, the various models tend to converge in terms of output.

After the global financial crisis of 2007-2009, more strict assessments of financial risk have been imposed by Basel III or Solvency II regulations. The backtesting literature has since gained traction after the development of the Unconditional Coverage (UC) backtest by Kupiec (1995), these include the works from Ziggel et al. (2014) and Wied et al. (2016).

In line with previous work, recent papers like Allen et al (2020) shows that even if we are using conditional volatility approach such as EWMA, the Gaussian approaches systematically underestimate tail risk. Moreover, they showed that the most widely used approach in practice, Historical Simulation, leads to too many consecutive VaR violations out-of-sample. This is problematic, as in times of market stress it may not be feasible to increase capital buffers. Khindanova and Rachev (2019) conclude that adequate approximation of distributional forms of returns is a key condition for accurate VAR estimation. Given the leptokurtic nature (heavy tails and excess kurtosis) of empirical financial data, the Pareto distributions seem to be the most appropriate distributional models for returns.

However, rather than presupposing the preeminence of any particular distribution, this study rigorously examines an expansive array of 61 theoretical distributions. The objective is not to validate the supremacy of Pareto distributions but to empirically determine which distribution most accurately encapsulates the behavior of financial returns and thereby enhances the precision of VaR estimates.

The investigation unveils a robust fitness methodology along with a versatile back-testing protocol that can be employed for any chosen distribution. The emphasis here is on the practicality and replicability of the methods, ensuring their accessibility and utility in diverse financial contexts. A case study analyzing the daily returns of a government security serves as the initial application of this methodology, yet the framework is designed with the flexibility to be adapted to a multitude of financial instruments.

Section 2 delineates the specifications of the VaR model, setting the stage for a deeper exploration. Section 3 details the employed methodology, capturing both the computation of VaR and the process for identifying and evaluating the theoretical distributions. The robustness of VaR calculations is also scrutinized within this part of the discourse. Section 4 is devoted to the presentation of the test results, demonstrating the efficacy of the distributions under examination. Finally, Section 5 culminates with a comprehensive summary of the findings and their implications for risk assessment in financial markets.

2. Model specifications

The huge popularity behind VaR is mainly due to its conceptual simplicity in aggregating the risk of a portfolio into a single number that can be easily used in board meetings, reported to regulators or presented to the public in an annual report. VaR can measure risk regardless of position, for almost any type of assets and risk factors (not just market risk) and offers a monetary and probabilistic expression of the possible loss.

Despite some problems, VaR can be used in several ways: (i) management can set a target risk and determine the appropriate position; (ii) VaR can be used to determine capital requirements, especially in the context of Basel and Solvency, but not only; (iii) VaR is used in reporting to the public; (iv) May be the basis of investment decisions. hedging, trading, and portfolio management; (v) it may be used as a rule in the remuneration of traders and managers. (Dowd, 2005)

The Bank for International Settlements (BIS) utilizes VaR in ascertaining capital buffers for financial market activities. The imperative for precise risk estimation is clear: inaccuracies can lead to either excess or deficient capital requirements, impacting the judicious allocation of financial resources.

VaR computation is diverse, with each methodology reflecting distinct presumptions, strengths, and limitations. Among these, traditional methods remain prevalent:

2.1. **Historical approach.** The most common nonparametric approach is historical simulation. In this approach it is considered that the historical distribution of the return is also relevant for its future distribution. VaR is estimated directly from the data without the need to derive parameters from other hypotheses. For example, considering 100 historical returns of the portfolio, VaR (95%) will be the value under the 5th lowest observation. However, this method assumes that returns are independently and identically distributed (IID), performing reliably only in the absence of significant volatility shifts. There are also different extensions of the model that assume an average weight with the volatility or the value of the most recent data.

2.2. **Parametric method (under a normal distribution).** This method considers that the return is normally distributed, and VaR is calculated using Gaussian distribution quantiles. The limitations of the method are obvious, since it is known that the return is not an IID, with many extreme values being recorded. The kurtosis of a financial series has values between 4 - 50, while the normal distribution implies a kurtosis of 3. (Cont, 2001; Kuesters et al, 2006)

VaR is computed as:

$$VaR(\alpha) = -\mu + \sigma * q\_z(\alpha)$$

where $q\_z(\alpha)$ is the value of standard normal distribution and μ and σ are the mean and standard deviation of the distribution.

2.3. **Monte-Carlo simulation.** The third method is similar to the historical method but involves the development of a model that simulates the future profitability of the asset. The hypothetical data are used to generate a theoretical distribution, not an empirical one. The main hypothesis assumes that the new distribution estimates or approximates the price behavior sufficiently well.

2.4. **Mathematical estimation.** This method is based on estimating Value at Risk using its mathematical definition. Given a confidence level $\alpha = (0.1)$ (usually 0.95 or 0.99) and defining an investment horizon T and a random variable L of the portfolio loss, VaR is defined as:

$$VaR_{\alpha}(L) := \inf\{l \in R : P(L \leq l) \geq \alpha\}$$

or

$$VaR_{\alpha}(L) = \min\{z | \; F_L(z) \geq \alpha\}$$

Among critics, Taleb (1997) suggests that (1) VaR has a dangerous potential and that it is often invalid in the real world. (2) Too much reliance on VaR can result in a very large loss. (2) VaR cannot determine losses over a certain probability. (3) It is not a coherent measure because it is not an additive measure, this being the most important limitation.

In addressing the limitations of the Value-at-Risk (VaR) methodology, it is also crucial to delineate the concepts of risk and uncertainty. Risk pertains to situations where the probabilities of outcomes are known and can be described by a probability distribution, allowing for quantifiable and often predictable risk management. Uncertainty, however, refers to the inability to predict outcomes due to unknown probabilities or the absence of information, which is not adequately captured by traditional VaR models. As highlighted by Alexander and Sarabia (2012), VaR can significantly underestimate the risk in the presence of model risk, which emerges from the reliance on potentially incorrect or overly simplistic assumptions, such as the normal distribution of financial returns. This misrepresentation leads to an underestimation of tail risk and a failure to account for extreme market events.

3. Data and Methodology

3.1. **Data description.** In the realm of finance, banks are prominent users of Value at Risk (VaR) methodologies, largely driven by regulatory imperatives to manage and report on the levels of risk within their portfolios. Bonds, as a staple component of bank portfolios, represent an area of significant interest for VaR analysis. The choice to focus on a bond for this analysis aligns with the objective to reflect the risk assessment practices within banking institutions where fixed-income securities such as bonds form a substantial part of the asset mix.

For this study, the chosen time series is based on the daily historical prices of a specific bond, RO1227DBN011, issued by the Romanian Ministry of Finance. This dataset, sourced from Thomson Reuters, spans from March 23, 2014, to October 23, 2020, offering a comprehensive view over various market conditions.

To ensure a thorough examination of the VaR model's effectiveness, the dataset will be divided into two distinct intervals: an in-sample period and an out-of-sample period. The in-sample segment will comprise 80% of the dataset, which equates to 1,317 observations, while the out-of-sample segment will contain the remaining 330 observations. This division allows for the assessment of the model's predictive accuracy and robustness by first establishing the model with the in-sample data and subsequently validating it against the out-of-sample data. The analyses will concentrate on the daily percentage change in the bond's price, acknowledging that while the study is centered on a bond, the methodology is versatile and can be applied to any financial asset to evaluate its risk profile.

3.2. **Methodology description.** The methodology employed in this study centers on calculating Value at Risk (VaR) with a focus on non-Gaussian distributions, diverging from the common assumption that financial returns follow a normal distribution. The initial step involves identifying a theoretical distribution that closely mirrors the empirical distribution of the data in question. To achieve this, the EasyFit (version 3.0) software was utilized, which applies the Kolmogorov-Smirnov, Anderson-Darling, and Chi-Squared tests to determine the most suitable fit. Out of 61 theoretical distributions tested, the top four, as determined by their fitness scores, were selected for further scrutiny.

To ensure the robustness of the selected distributions, fitness tests were conducted for both in-sample and out-of-sample periods. For the latter, an adjustment was made where the earliest 330 observations were excluded, and the subsequent out-of-sample observations were incorporated, ensuring that the comparison was based on an equivalent data set of 1317 points for each distribution.

Once the theoretical distribution was ascertained, the next step involved calculating the VaR for the out-of-sample period. This was achieved by employing the probability density function (pdf) parameters, which were computed using the scipy.stats library (v1.11.3) within the Python (3.6) programming environment.

With a specified confidence level—95% and 99%—the study proceeded to calculate VaR through numerical approximation of the integral, facilitated by functions available in the scipy. integrate library (v1.11.3). The VaR value at which the integrated area of the density function corresponds to the desired confidence level was determined using the scipy.optimize.fsolve function (v1.11.3), which identifies the minimum value where the cumulative density function equals or exceeds the given probability threshold.

For the purposes of this research, VaR was calculated at five-day intervals, employing a rolling window of the most recent 250 observations, and considering a standardized investment of one monetary unit. The validity of the calculated VaR figures was further tested through a rigorous back-testing methodology, thereby reinforcing the credibility of the findings.

3.3. **Back - Testing.** To evaluate the accuracy of the VaR model presented in this paper, three tests will be performed to examine whether the model is correctly specified or not. To determine if the selected distribution is indeed robust, we will use the back-testing procedure for out of sample data. A starting point is a so-called indicator function:

$$I_t = \begin{cases} 1,\ if\ r_t \leq VaR_t \\ 0,\ if\ r_t > Var_t \end{cases}$$

In order to be considered correct, the indicator function must fulfill two properties presented by Christoffersen (1998):

(1) the unconditional coverage property assumes that I_t takes the value of 0 exactly (1-α) * 100 in cases. If it is 0 less times, the model is too conservative.
(2) the independence property assumes that two consecutive elements of the indicator must be independent of each other. If the condition is not met, the model will not be very to catch the changes in the market. According to Campbell (2005), I_t must be iid with a Bernoulli random variable with probability p.

The first test is the Kupiec test (1995) for unconditional coverage. It tests that the maximum likelihood function is asymptotically distributed $\chi^2(1)$. Once defined I_t, the likelihood function is:

$$L\left(p; I_1, I_2, \ldots, I_T\right) = (1-p)^{n_0} p^{n_1}$$

Where p is the probability under which we calculate VaR (ex 0.95%).We define $\hat{p} = \frac{n_1}{n_0+n_1}$ to be the empirical probability obtained (Christoffersen, 1998, p. 845). The number of cases in which the restriction of $r_t \leq VaR_t$ was not violated (or violated) is given by n_1 (n_0). Testing for unconditional coverage is done through a simple log-likelihood ratio:

$$LR_{UC} = -2\ln\left(\frac{L\left(p; I_1, .., I_n\right)}{L\left(\hat{p}; I_1, .., I_n\right)}\right) = -2\ln\left(\frac{(1-p)^{n_0} p^{n_1}}{(1-\hat{p})^{n_0} \hat{p}^{n_1}}\right) \sim \chi^2(1)$$

Under the null hypothesis, LR is in the interval:

$$chiinv\left(1-\frac{p}{2}; 1\right) \leq LR_{UC} \leq chiinv\left(\frac{p}{2}; 1\right)$$

Kupiec's test is not sufficient to explain whether the breach cases are id or grouped. Christoffersen (1998) suggests a plausibility test for both unconditional coverage (UC), but also for the value of independence (ind) and conditional coverage (CC). The UC test is similar to Kupiec's.

$$LR_{ind} = -2\ln\left(\frac{(1-\pi_2)^{n_{00}+n_{11}} \pi_2^{n_{01}+n_{11}}}{(1-\pi_{01})^{n_{00}} \pi_{01}^{n_{11}} (1-\pi_{11})^{n_{10}} \pi_{11}^{n_{11}}}\right) \sim \chi^2(1)$$

$$where\ \pi_{01} = \frac{n_{01}}{n_{00}+n_{01}}; \pi_{11} = \frac{n_{11}}{n_{01}+n_{11}}; \pi_2 = \frac{n_{01}+n_{11}}{n_{00}+n_{10}+n_{01}+n_{11}}$$

Here, n_{ij} is the number of observations with the value I at $t-1$ followed by j at the moment t (1 if the relationship is broken and 0 if the loss is smaller than VaR). Christofferson (1998).

Christofferson (1998) demonstrates that ignoring the first observation there is a numerical relationship between LR_{UC} *si* LR_{ind}. In his work he also shows that the distribution of the conditional coverage test is asymptotic χ^2 with 2 degrees of freedom:

$$LR_{CC} = LR_{UC} + LR_{ind} \ \sim \ \chi^2(2)$$

Thus, H_0 is not rejected for a significance degree $p = 1 - \alpha$ if:

$$chiinv\left(1 - \frac{p}{2}; 2\right) \leq LR_{CC} \leq chiinv(\frac{p}{2}; 2)$$

This test is used in the present paper as the main indicator in testing the adequacy of the model.

4. Results

4.1. **Fitting.** The results presented in Figures 1 and 2, along with Table 1, encompass the outcome of fitting of the best four statistical distributions (out of 61) to empirical data for both in-sample and out-of-sample periods. These figures reveal the probability density functions overlaid on the actual data distributions. In the case of the in-sample period, as depicted in Figure 1, the Cauchy distribution exhibits the closest alignment with the empirical data, indicating its superior fit over other distributions such as Laplace, Logistic, and Beta. The empirical distribution's left tail mass and the significant tails of the Cauchy distribution are particularly noteworthy, suggesting a pronounced presence of outliers or extreme values in the data set.

Table 1 - Goodness of fit score for the best 4 distributions

			In sample	**Out-of-sample**
Cauchy	**Kolmogorov-Smirnov**	Statistic	0.03	0.04
		P-Value	0.08	0.02
	Anderson-Darling	Statistic	4.27	5.03
	Chi-squared	Statistic	39.56	36.66
		P-Value	0.00	0.00
Laplace	**Kolmogorov-Smirnov**	Statistic	0.06	0.05
		P-Value	0.00	0.00
	Anderson-Darling	Statistic	10.64	5.66
	Chi-squared	Statistic	75.36	34.80
		P-Value	0.00	0.00
Logistic	**Kolmogorov-Smirnov**	Statistic	0.10	0.09
		P-Value	0.00	0.00
	Anderson-Darling	Statistic	30.22	20.14
	Chi-squared	Statistic	255.00	169.45
		P-Value	0.00	0.00
Beta	**Kolmogorov-Smirnov**	Statistic	0.12	0.11
		P-Value	0.00	0.00
	Anderson-Darling	Statistic	44.80	32.30
	Chi-squared	Statistic	370.61	259.36
		P-Value	0.00	0.00

The out-of-sample period, illustrated in Figure 2, continues to show the Cauchy distribution as the best fit among the chosen models. This is further corroborated by Table 1, where goodness-of-fit tests—namely, the Kolmogorov-Smirnov, Anderson-Darling, and Chi-squared tests—are detailed. Across these tests, we notice that while all distributions pass the goodness-of-fit criteria, the Cauchy distribution generally presents the lowest statistics (indicating a better fit) and acceptable p-values, especially notable at the 95% confidence level. This trend suggests that while other distributions may still provide a reasonable model of the data, the Cauchy distribution consistently emerges as the best descriptor of the empirical distribution's behavior.

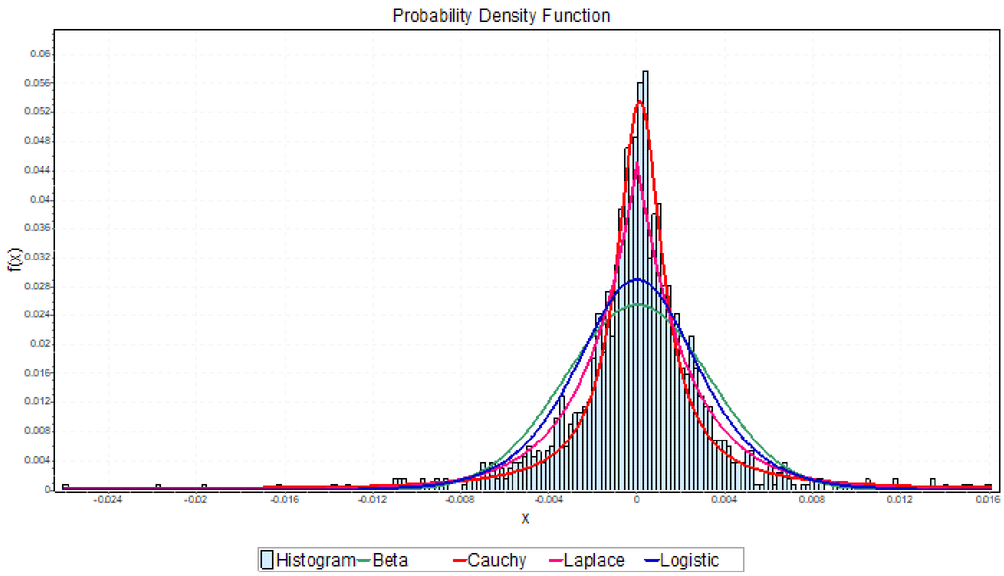

Figure 1 - Probability density function for the in-sample period

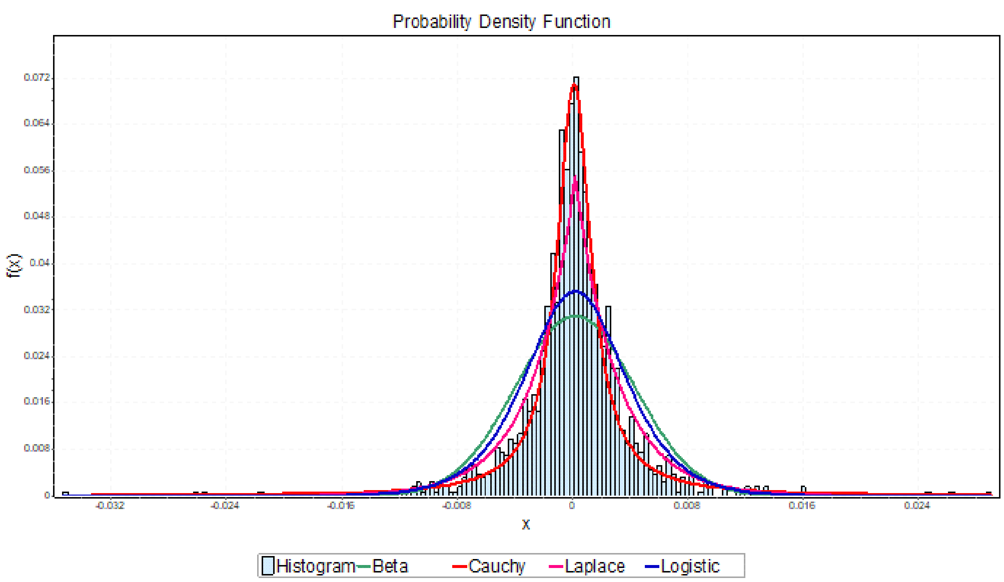

Figure 2 - Probability density function for the out-of-sample period

Interestingly, despite the robust fitting of the Cauchy distribution within the body of the data, Figure 3's Q-Q plot elucidates that it does not effectively capture the tail events. The Cauchy distribution overestimates both the potential maximum losses and profits, leading to the anticipation of a higher Value at Risk (VaR) when this distribution is employed. This aspect is crucial for risk management purposes, as it implies that using the Cauchy distribution for VaR calculations could result in more conservative risk estimates, potentially leading to higher capital reserves or reduced risk-taking than might be justified by the empirical data.

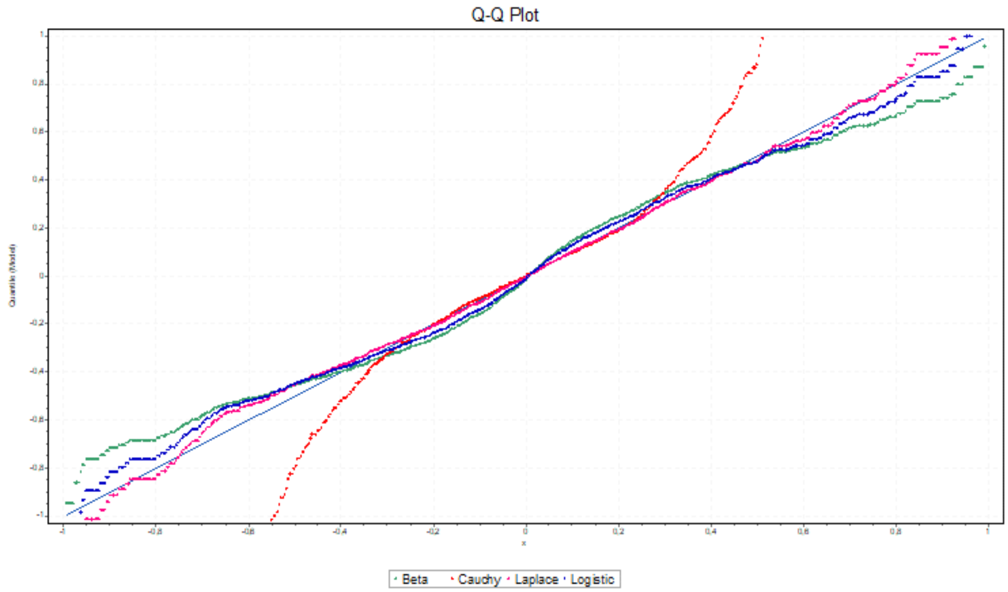

Figure 3 - Q-Q Plot for the selected distributions

4.2. **Back-Testing.** Back-testing results are shown in the Table 2. Cauchy and Laplace distributions pass the Kupiec test for both 95% and 99% probability and Logistic, and Beta distributions only for 95%. However, none of the distributions pass the Christoffersen test because the value of independence test (LR ind) is too high. This means that VaR is not independent and does not have an identical distribution through time, being an inefficient measure in the case of a big disruption in the market.

Table 2 - Backtesting results for selected distributions

Distrib.	Prob.	Kupiec test			Indep. test	Christoffersen test		
		Confidence interval bound		LR(UC)	LR(Ind)	Confidence interval bound		LR(CC)
		lower	upper			lower	upper	
Cauchy	95%	0.000982	5.023886	4.2675	43.5830	0.050635	7.377758	47.8506
	99%	0.000392	7.879438	6.6332	23.1250	0.010025	10.59663	30.7560
Laplace	95%	0.000982	5.023886	2.4138	77.5180	0.050635	7.377758	79.9318
	99%	0.000392	7.879438	4.8362	38.2695	0.010025	10.59663	43.1058
Logistic	95%	0.000982	5.023886	4.0083	83.6510	0.050635	7.377758	87.6593
	99%	0.000392	7.879438	16.5375	54.4794	0.010025	10.59663	71.0170
Beta	95%	0.000982	5.023886	0.3809	63.9379	0.050635	7.377758	64.3188
	99%	0.000392	7.879438	11.2703	53.4783	0.010025	10.59663	64.7486

Since the other distributions tested do not capture very well the events with low probability of occurrence, as comparison, the Value at Risk was calculated using also extreme value theory (EVT). This model the extreme events in the distribution tails ($> 95\%$) using a Generalized Pareto distribution. The value at risk thus calculated is an improvement of the historical VaR, much better surprising the events in the tails. A comprehensive description of the calculation method can be found in Fabozzi (2015).

4.3. **Evolution of Value at Risk.**

Table 3 - Evoluton of VaR in the out-of-sample period

Value at risk was recalculated every 5 days using a 250-day rolling window. One can easily observe (Table 3) the very conservative behavior of the Cauchy distribution identified in the case of QQ-plot. Its very long tails determine a VaR calculated at the probability of 99% up to 5.7 times higher than for the other distributions. The Cauchy distribution's conservative nature at the 99% level is notable when compared to its 95% level estimates, which are much less conservative and more in line with other distributions.

Additionally, the Extreme Value Theory (EVT) method also produces a conservative VaR estimate, albeit not as extreme as the Cauchy distribution. The EVT's VaR at the 99% confidence level is noted to be up to three times higher than the historical VaR. This conservative estimation is crucial because EVT is designed to assess risk based on the extreme values within a dataset, which are often the focus of risk management practices.

Most other distributions, including Laplace, Logistic, and Beta, along with the historical method, show their 99% VaR estimates within a narrow range (0.6% - 0.8%). The proximity of these values suggests that, aside from the Cauchy and EVT methods, there is little variability between the different distribution methods at the 99% confidence level. At the 95% confidence level, there is an even smaller difference between the VaR estimates of various distributions, as shown by their minimal variances. This suggests that for less extreme risk assessments, the choice of distribution may have a smaller impact on the VaR estimate. The historical method's variance at both the 99% and 95% levels is higher than most parametric methods, indicating more fluctuation in its VaR estimates. This could be due to the historical method's direct reliance on past data, which can be highly variable.

5. Conclusions

In this study, we explored the computation of Value at Risk (VaR) for a financial asset with non-normal return distributions. By leveraging EasyFit software, 61 theoretical distributions were fitted to empirical data, with the Cauchy, Laplace, Logistic, and Beta distributions emerging as the most suitable based on various goodness-of-fit tests including Kolmogorov-Smirnov, Anderson-Darling, and Chi-squared.

Our analysis extended to the assessment of VaR robustness via Back-Testing methods, applying both Kupiec and Christoffersen tests. It was observed that while most distributions satisfactorily passed the Kupiec unconditional coverage test at 95% and 99% confidence levels, the Christoffersen test of independence posed a significant challenge, with none of the distributions meeting the stringent criteria. This suggests that the independence test may be overly stringent, pointing to a need for further evaluation of its applicability in practical risk management contexts.

A notable finding was the conservatively high VaR results calculated using the Cauchy distribution, which were as much as 5.7 times greater than those estimated by other distributions, emphasizing the Cauchy distribution's sensitivity to tail events. In contrast, VaR values estimated by the Extreme Value Theory (EVT) were found to be up to threefold higher than those derived from historical methods. The other distributions examined provided VaR estimates closely aligned with historical VaR, underscoring a potential preference for simpler, more traditional approaches in certain risk assessment scenarios.

Given the diversity of results observed across different computation methods, it's apparent that a one-size-fits-all approach to VaR estimation may not be feasible. The findings of this study intersect with the literature in several key areas. Allen et al. (2020) emphasize the underestimation of tail risk in Gaussian approaches and the inadequacies of Historical Simulation, which our analysis confirms. Similarly, our observations resonate with the concerns raised by Khindanova and Rachev (2019) regarding the criticality of selecting appropriate distributional models to achieve accurate VaR estimations. However, this study diverges from the conventional presumption of the superiority of any given distribution, such as Pareto distribution. Notably, the Pareto distribution did not rank among the top four distributions that were identified as the best fits and selected for more in-depth analysis.

Looking forward, this research could be expanded to encompass comparative analyses across various classes of financial assets. Such studies could ascertain the relative merits of non-Gaussian distribution models over traditional methods for VaR calculations. Also, more analysis could suggest a distribution that may be satisfactory applied to most financial assets. Meanwhile, the significant variations in VaR estimates identified in this study indicate a pressing need for the finance community to refine risk assessment models, ensuring that risk managers have at their disposal the most reliable and accurate tools for managing financial risk.

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